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100

$$1 \quad f(e^x - 1) = e^{2x} - e^x - x = f(x).$$

$$e^x - 1 = u \quad x = \ln(u + 1)$$

$$f(u) = (u + 1)^2 - (u + 1) - \ln(u + 1) = u^2 + u - \ln(u + 1)$$

$$f'(x) = 2x - x - \ln(x)$$

$$2 \quad \lim_{x \rightarrow 0} \frac{\sin x - \sin \sin x - \sin x}{x^4}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(\sin x - \sin \sin x) \sin x}{x^4} &= \lim_{x \rightarrow 0} \frac{\sin x - \sin \sin x}{x^3} \cdot \lim_{x \rightarrow 0} \frac{\cos x - \cos(\sin x) - \cos x}{3x^2} \\ &= \lim_{x \rightarrow 0} \frac{\cos x (1 - \cos(\sin x))}{3x^2} \cdot \lim_{x \rightarrow 0} \frac{\sin(\sin x) - \cos x}{6x} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \frac{1}{6} \end{aligned}$$

$$3 \quad \lim_n \frac{3^{n-1} - 2^n}{2^{n-1} - 3^n}$$

$$\lim_n \frac{3^{\frac{2}{3^n}}}{2^{\frac{2}{3^n}}}$$

$$\lim_n |r|^n = 0$$

4

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1-x} - \sqrt[3]{1+x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{1-x}{\sqrt{1-x}} - \frac{1+x}{\sqrt{1+x}} = \frac{2}{2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - 1}{x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2}x - \frac{x}{2}}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{1-x}} - \frac{1}{2\sqrt{1+x}}}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{1-x}{x^2} - \frac{1+x}{x^2} = \frac{2}{3}$$

$$\frac{1}{1}$$

$$\lim_n \left(1 - \frac{1}{2^2} - 1 - \frac{1}{3^2} - \dots - 1 - \frac{1}{n^2} \right)$$

$$\lim_n \left(1 - \frac{1}{2} - 1 - \frac{1}{2} - 1 - \frac{1}{3} - 1 - \frac{1}{3} - \dots - 1 - \frac{1}{n} - 1 - \frac{1}{n} \right)$$

$$\lim_n \left(\frac{1}{2} - \frac{3}{2} - \frac{2}{3} - \frac{4}{3} - \dots - \frac{n}{n} - \frac{1}{n} - \frac{n}{n} \right) = \lim_n \frac{n-1}{2n} = \frac{1}{2}$$

5

$$\lim_x \left(1 - \frac{2}{x} \right)^{x-10} = \lim_{x \rightarrow 0} \frac{1-x}{1+x}^{\frac{1}{x}}$$

$$\lim_n \left(1 - \frac{2}{x} \right)^{x-10} = \lim_x \left(1 - \frac{2}{x} \right)^{\frac{x-10}{x}}$$

$$\lim_x \left(1 - \frac{2}{x} \right)^{\frac{x}{2} - 1} = e^{-2}$$

$$\lim_{x \rightarrow 0} \frac{1}{1-x} x^{\frac{1}{x}} = \frac{\lim_{x \rightarrow 0} 1}{\lim_{x \rightarrow 0} \frac{1}{x}} x^{\frac{1}{x}} = \frac{\lim_{x \rightarrow 0} 1}{e} x^{\frac{1}{x}} = \frac{1}{e} x^{\frac{1}{x}}$$

$$\lim_{x \rightarrow 0} \frac{1-x}{1+x}^{\frac{1}{x}} \quad \lim_{x \rightarrow 0} \frac{1-x-2x}{1+x}^{\frac{1}{x}} \quad \lim_{x \rightarrow 0} 1 - \frac{2x}{1+x}^{\frac{1}{x} - \frac{2}{1-x}} e^2$$

6

$$1 \quad \lim_{x \rightarrow 0} (1 - \tan x)^{\cot x} \qquad \lim_{x \rightarrow 1} x^{\frac{4}{x-1}}$$

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x}$$

$$\tan x = t \quad \cot x = \frac{1}{t} \quad x = 0 \quad t = 0$$

$$\lim_{x \rightarrow 0} (1 + \tan x)^{c \cot x} = \lim_{t \rightarrow 0} (1 + t)^{\frac{1}{t}} = e$$

$$\lim_{x \rightarrow 1} x^{\frac{4}{x-1}} = \lim_{t \rightarrow 0} (1-t)^{\frac{4}{t}} = \lim_{t \rightarrow 0} (1-t)^{\frac{1}{t}}{}^4 = e^4$$

$$\lim_{x \rightarrow 0} (\cos x)^{\cot^2 x} = \lim_{x \rightarrow 0} (1 - \sin^2 x)^{\frac{\cos^2 x}{2 \sin^2 x}} = \lim_{x \rightarrow 0} 1 - (\sin^2 x)^{\frac{1}{2}} = 1 - \frac{1}{2}$$

7

$$1 \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 - k}} \quad \lim_{n \rightarrow \infty} \frac{k}{n^2 - k}$$

$$1 \quad \frac{n}{\sqrt{n^2}} \quad \frac{n}{\sqrt{n^2}} \quad \frac{1}{\sqrt{n^2}} \quad \frac{n}{\sqrt{n^2}}$$

$$\begin{array}{c} \text{li } m \\ \sqrt{n^2} \end{array} \quad \begin{array}{c} n \\ n \end{array} \quad \begin{array}{c} \text{li } m \\ \sqrt{1} \end{array} \quad \begin{array}{c} 1 \\ \frac{1}{n} \end{array}$$

$$\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + 1}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{n^2}}} = 1$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + k}} = 1$$

$$\frac{1 \ 2 \ \dots \ n}{n^2 \ n \ n} \quad \quad \quad \frac{n \quad k}{n^2 \ n \ k} \quad \quad \quad \frac{1 \ 2 \ \dots \ n}{n^2 \ n \ 1}$$

$$\frac{1}{n} \left(1 + \frac{1}{n} + \frac{2}{n^2} + \dots + \frac{n}{2n} \right) = \frac{1}{n} \left(1 + \frac{1}{n} + \frac{2}{n^2} + \dots + \frac{n}{2n} \right) = \frac{1}{n} \left(1 + \frac{1}{n} + \frac{2}{n^2} + \dots + \frac{n}{2n} \right)$$

$$\lim_n \frac{1}{n^2} \frac{2}{n} \cdots \frac{n}{n-1} = \lim_n \frac{\frac{1}{2} n(n-1)}{n^2} = \frac{1}{2}$$

$$\lim_n \frac{n}{k+1} \frac{k}{n-k} = \frac{1}{2}$$

$$8 \quad \lim_n \frac{n}{k+1} \frac{n}{n^2 k^2}$$

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$$\frac{n^2}{n^2} \frac{n}{n^2} \frac{n}{k^2} \frac{n^2}{n^2} \frac{1}{1^2}$$

$$\lim_n \frac{n^2}{n^2} \frac{1}{n^2} = \frac{1}{2} \quad \lim_n \frac{n^2}{n^2} \frac{1}{1^2} = 1$$

$$\lim_n \frac{n}{k+1} \frac{n}{n^2 k^2} = \lim_n \frac{1}{n} \frac{n}{k+1} \frac{1}{k^2}$$

$$\frac{1}{0} \frac{dx}{1-x^2} = \arctan x \Big|_0^1 = \frac{\pi}{4}$$

$$9 \quad \lim_n \frac{\frac{1}{n} \sin \frac{1}{n}}{\sin^3 \frac{1}{n}}$$

$$\lim_{x \rightarrow 0} \frac{x \sin x}{\sin^3 x} = \lim_{x \rightarrow 0} \frac{x \sin x}{x^3}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \frac{1}{6}$$

$$10 \quad \lim_{x \rightarrow 0} \frac{e^{\frac{1}{x^2}}}{x^{10}}$$

$$\frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{\frac{2}{x^3} e^{\frac{1}{x^2}}}{10x^9} = \lim_{x \rightarrow 0} \frac{e^{\frac{1}{x^2}}}{5x^{12}}$$

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$$\frac{1}{x^2} t$$

$$\lim_{x \rightarrow 0} \frac{e^{\frac{1}{x^2}}}{x^{10}} = \lim_{t \rightarrow 0} \frac{e^t}{t^5} = \frac{5}{t^4} = \infty$$

$$\lim_t \frac{5t^4}{e^t} = \cdots = \lim_t \frac{5!}{e^t} = 0$$

$$11 \quad \lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{e^x - 1}.$$

$$\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{e^x - 1} = \lim_{x \rightarrow 0} \frac{(e^x - 1) - x}{x(e^x - 1)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{(e^x - 1) - xe^x} = \lim_{x \rightarrow 0} \frac{e^x}{e^x - e^x - xe^x}$$

$$\lim_{x \rightarrow 0} \frac{1}{2 - x} = \frac{1}{2}$$

$$12 \quad \lim_{x \rightarrow 0} \left(\frac{1}{\sin^2 x} - \frac{\cos^2 x}{x^2} \right)$$

$$\lim_{x \rightarrow 0} \frac{x^2 - \sin^2 x \cos^2 x}{x^2 \sin^2 x}$$

$$\lim_{x \rightarrow 0} \frac{x^2 - \frac{1}{4} \sin^2 2x}{x^4}$$

$$\lim_{x \rightarrow 0} \frac{2x - \frac{4}{4} \sin 2x \cos 2x}{4x^3}$$

$$\lim_{x \rightarrow 0} \frac{x - \frac{1}{4} \sin 4x}{2x^3}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{6x^2} = \lim_{x \rightarrow 0} \frac{4 \sin 4x}{12x} = \frac{4}{3}$$

$$13 \quad f(x) = \begin{cases} x^2 - 1, & |x| < c \\ \frac{2}{|x|}, & |x| \geq c \end{cases} \quad (c > 0) \quad c = \underline{\hspace{2cm}}.$$

1

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} f(x) = c^2 - 1 = \frac{2}{c} - 1 \quad c = 1$$

$$14 \quad \lim_{x \rightarrow 0} x^{\sin^2 x}$$

$$y = x^{\sin^2 x} \quad \ln y = \sin^2 x \ln x$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \sin^2 x \ln x = 0$$

$$\lim_{x \rightarrow 0} y = e^0 = 1$$

$$15 \quad \lim_{x \rightarrow 0} \cos x^{\cot^2 x}$$

$$y = \cos x^{\cot^2 x} \quad \ln y = \cot^2 x \ln \cos x$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \cot^2 x \ln \cos x = \lim_{x \rightarrow 0} \frac{\ln \cos x}{\tan^2 x} = \lim_{x \rightarrow 0} \frac{\ln \cos x}{x^2}$$

$$\frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{\tan x}{2x} \quad \frac{1}{2} \quad \lim_{x \rightarrow 0} y = e^{\frac{1}{2}}$$

$$16 \quad \lim_{x \rightarrow 0} \sin \frac{1}{x} \cos \frac{1}{x}$$

$$\ln y = x \ln \sin \frac{1}{x} \cos \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{\ln \sin \frac{1}{x} \cos \frac{1}{x}}{\frac{1}{x}} = \lim_{t \rightarrow 0} \frac{\ln(\sin t \cos t)}{t}$$

$$\lim_{t \rightarrow 0} \frac{\cos t - \sin t}{\sin t - \cos t} = 1$$

$$\lim_{x \rightarrow 0} y = e$$

$$17 \quad \lim_{x \rightarrow 0} \frac{1}{x^2} \ln \frac{\sin x}{x}$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} \ln \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{1}{x^2} \ln \left(1 - \frac{\sin x}{x} \right) = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \frac{1}{6}$$

$$18 \quad \lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \arctan 3x}{(e^x - 1) \ln(1 - 2x) \sin 5x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(2x)^2 - (3x)}{x[(2x) - (5x)]} = \frac{3}{5}$$

$$19 \quad \lim_{x \rightarrow 0} \frac{3 \sin x - x^2 \cos \frac{1}{x}}{(1 - \cos x) \ln(1 - x)}$$

$$\frac{0}{0}$$

$$20 \quad \lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - o(x^5) \quad x \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{\frac{x^5}{5!} + o(x^5)}{x^5} = \frac{1}{5!} = \frac{1}{120}$$

$$21 \quad f(x_0) = 2 \lim_{x \rightarrow 0} \frac{f(x_0 + 3x) - f(x_0 + 2x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{f(x_0 + 3x) - f(x_0) - f(x_0 + 2x) + f(x_0)}{x}$$

$$3 \lim_{x \rightarrow 0} \frac{f(x_0 + 3x) - f(x_0)}{3x} - 2 \lim_{x \rightarrow 0} \frac{f(x_0 + 2x) - f(x_0)}{2x}$$

$$3f(x_0) - 2f(x_0) - 5f(x_0) = -10f(x_0)$$

$$22 \quad y = f(x) = y = \sin x \quad \lim_{n \rightarrow \infty} n f\left(\frac{2}{n}\right)$$

$$f(0) = 0 \quad f'(0) = (\sin x)|_{x=0} = 1$$

$$\lim_{n \rightarrow \infty} n f\left(\frac{2}{n}\right) = \lim_{n \rightarrow \infty} \frac{f\left(\frac{2}{n}\right) - f(0)}{\frac{2}{n} - 0} = f'(0) = 2$$

$$23 \quad a > 0, x_1 = b > 0, x_2 = \frac{1}{2}x_1 + \frac{a}{x_1}, \quad x_n = \frac{1}{2}x_{n-1} + \frac{a}{x_{n-1}}, \quad \lim_{n \rightarrow \infty} x_n$$

$$x_n = \sqrt{x_{n-1} \left(\frac{a}{x_{n-1}} + x_{n-1} \right)} = \sqrt{a + x_{n-1}^2} \quad \wedge$$

$$x_{n+1} - x_n = \frac{1}{2}x_n + \frac{a}{x_n} - x_n = \frac{a - x_n^2}{2x_n} = 0 \quad x_{n+1} = x_n$$

$$x_n = 1 \quad \lim_{n \rightarrow \infty} x_n = A$$

$$x_n = \frac{1}{2}x_{n-1} + \frac{a}{x_{n-1}} \quad A = \frac{1}{2}A + \frac{a}{A}$$

$$A^2 = a \quad A > 0 \quad A = \sqrt{a} \quad \lim_{n \rightarrow \infty} x_n = \sqrt{a}$$

24

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & x < 0 \\ \frac{x^2}{1 - \cos x} & x > 0 \end{cases}$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} 2 \frac{\sin 2x}{2x} = 2$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{x^2}{\frac{1}{2}x^2} = 2$$

$$\lim_{x \rightarrow 0} f(x) = 2$$

$$25 \quad \lim_{x \rightarrow 0} \frac{2 - \frac{1}{e^x}}{1 - \frac{4}{e^x}} \frac{\sin x}{|x|}$$

$$\lim_{x \rightarrow 0} \frac{2 - \frac{1}{e^x}}{1 - \frac{4}{e^x}} \frac{\sin x}{(x)} = 2 - 1 = 1$$

$$\lim_{x \rightarrow 0} \frac{2e^{\frac{4}{x}} - e^{\frac{3}{x}}}{e^{\frac{4}{x}} - 1} \frac{\sin x}{x} = 0 - 1 = -1$$

$$\lim_{x \rightarrow 0} \frac{2 - \frac{1}{e^x}}{1 - \frac{4}{e^x}} \frac{\sin x}{|x|} = 1$$

$$26 \quad \lim_{x \rightarrow 1} \frac{x^2 - ax - b}{\sin(x^2 - 1)} = 3 \quad a = b.$$

$$\lim_{x \rightarrow 1} (x^2 - ax - b) = 0$$

$$\lim_{x \rightarrow 1} \frac{x^2 - ax - b}{\sin(x^2 - 1)} = \lim_{x \rightarrow 1} \frac{2x - a}{2x \cos(x^2 - 1)} = \frac{2 - a}{2} = 3 \quad a = 4, b = 5$$

$$27 \quad f(x) = \lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{(e^{x^2} - 1)f(x)} = 1 \quad f(0) = \frac{1}{2}$$

$$\frac{1}{2}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2} \sin^2 x}{x^2 f(x)} = 1, \quad \lim_{x \rightarrow 0} \frac{\frac{1}{2}}{f(x)} = 1 \quad f(x) = f(0) = \frac{1}{2}$$

$$28$$

$$\frac{1}{0} = 0$$

$$0 = 0$$

$$0 = 0$$

$$x \rightarrow 0$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

$$f(0) = 0$$

$$f(0) = 0 \quad f(0) = 0 \quad f(0) = 0 \quad f(x) = x \quad x = 0$$

29

$$f(x) = \begin{cases} \frac{\ln(1-x)}{x} & x < 0 \\ \frac{1}{2} & x = 0 \\ \frac{\sqrt{1+x} - 1}{x} & x > 0 \end{cases}$$

$$x = 0$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{\ln(1-x)}{x} = \lim_{x \rightarrow 0} \ln(1-x)^{\frac{1}{x}} = 1$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1} = \frac{1}{2}$$

$$f(0) = 0 \quad f(0) = 0 \quad \lim_{x \rightarrow 0} f(x) = f(x) \quad x = 0$$

$$30 \quad f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ k & x = 0 \end{cases} \quad k.$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$f(0) = k \quad k = 1$$

$$31 \quad f(x) = \frac{\sqrt[3]{x} - 1}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt[3]{x} - 1} = 1$$

$$\lim_{x \rightarrow 1} \frac{1}{\sqrt[3]{x^2} - \sqrt[3]{x} + 1} = \frac{1}{3}$$

$$x = 1 \quad f(x) = \frac{1}{3}$$

$$32 \quad f(x) = \frac{x^2 - 2x}{|x| - x^2 - 4}$$

$$x = 0 \quad -2 \quad 2$$

$$x = 0 \quad -2 \quad 2$$

$$x = 0$$

$$f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{x(x-2)}{x(x-2)(x+2)} = \frac{1}{2} \quad f(0) = 0 \quad \lim_{x \rightarrow 0} \frac{x(x-2)}{x(x-2)(x+2)} = \frac{1}{2}$$

$$x = 0$$

$$x = 2$$

$$f(x-2) - f(x-2) = \lim_{x \rightarrow 2} \frac{x(x-2)}{|x|(x-2)(x-2)}$$

$$\frac{x-2}{x-2} = \frac{1}{x-2}$$

$$f(2-0) - f(2-0) = \lim_{x \rightarrow 2} \frac{x(x-2)}{x(x-2)(x-2)} = \frac{1}{4}$$

$$\frac{x-2}{x-2} = \frac{1}{x-2} \quad f(2) = \frac{1}{4} \quad f(x) = \frac{1}{x-2}$$

$$33 \quad f(x) = (x, y) \quad \lim_{x \rightarrow a} f(x) = a$$

$$g(x) = \begin{cases} f\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$36 \quad f(x) \in [a, b] \quad f(a) = a \quad f(b) = b \quad f(x) = x \quad (a, b)$$

$$g(x) = f(x) - x \quad g(x) \in [a, b]$$

$$g(a) = f(a) - a = 0$$

$$g(b) = f(b) - b = 0$$

$$g(x) \in (a, b) \quad f(x) = x \quad (a, b)$$

$$37 \quad e^x - e^{-x} - 4 \cos x \in (0, 2)$$

$$f(x) = e^x - e^{-x} - 4 \cos x - 4 \quad f(x) \in (0, 2)$$

$$f(0) = 3 > 0 \quad 2^2 = 2^2$$

$$f(x) \in (0, 2) \quad (0, 2) \quad f(x) = 0$$

$$f(x) = e^x - e^{-x} - \sin x - 0 \quad x = 0 \quad f(x) \in (0, 2) \quad \text{别} \quad f(x) \in (0, 2)$$

$$f(x) \in (0, 2) \quad f(x) \in (0, 2)$$

$$(0, 2)$$

$$38 \quad f(x) = x - a \quad g(x) = g(x) - a \quad f(a)$$

$$\therefore g(x)$$

$$f(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow a} \frac{x - a - g(x)}{x - a} = 0$$

$$= \lim_{x \rightarrow a} \frac{g(x)}{x - a}$$

$$39 \quad \sin xy - \ln y - x = 0, 1 \quad \underline{\hspace{2cm}}.$$

$$y = x - 1.$$

$$F(x, y) = \sin(xy) - \ln(y - x) - x = k \quad \frac{F_x}{F_y} = \frac{y \cos(xy) - \frac{1}{y-x} - 1}{x \cos(xy) - \frac{1}{y-x}} \quad (0, 1) \quad k = 1$$

$$y = 1 - x \quad y = x - 1$$

$$40$$

$$y = f(x) = |x| \quad \begin{matrix} x & x & 0 \\ x & x & 0 \end{matrix}$$

$$x_0 = 0$$

$$y = f(x) = |x| \quad x_0 = 0 \quad f(0) = 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f(x) = 0$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x = 0$$

$$\lim_{x \rightarrow 0} |x| = f(0) = 0$$

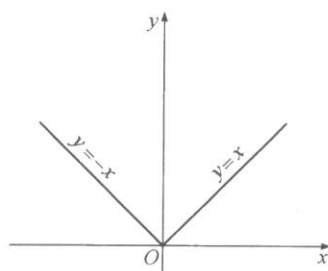
$$x_0 = 0 \quad f(x)$$

$$f(0) = \lim_{x \rightarrow 0} \frac{y}{x} = \lim_{x \rightarrow 0} \frac{|0 - x| - 0}{x}$$

$$\lim_{x \rightarrow 0} \frac{|x|}{x} = \lim_{x \rightarrow 0} \frac{x}{x} = 1$$

$$f(0) = \lim_{x \rightarrow 0} \frac{y}{x} = \lim_{x \rightarrow 0} \frac{|0 - x| - 0}{x}$$

$$\lim_{x \rightarrow 0} \frac{|x|}{x} = \lim_{x \rightarrow 0} \frac{x}{x} = 1$$



$$f(0) = f(0) = y = |x|$$

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$$f(x) = \frac{x^2}{ax + b} \quad x \neq 1$$

$$a \neq b \quad f(x) = \frac{x-1}{x-1}$$

$$\therefore f(x) = \frac{x-1}{x-1}$$

$$f(1) = 0 \quad \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} x^2 = 1$$

$$f(x) = \frac{x-1}{x-1} \quad a \neq 1 \quad b \neq 1 \quad a$$

$$f(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} (x + 1) = 2$$

$$f(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{ax + b - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{a(x - 1) + a + b - 1}{x - 1} = a$$

$$f(x) = x - 1 \quad f(1) = f(1) = 2 - a$$

$$a - 2 = b - 1 \quad a - 1 = 2 - 1 \quad f(x) = x - 1$$

42

$$y = \sqrt{1 - x^2} \ln(x + \sqrt{1 - x^2}) \quad y = \cot^2 x + \arccos \sqrt{1 - x^2}$$

$$y = \sqrt{1 - x^2} \ln x + \sqrt{1 - x^2} = \sqrt{1 - x^2} \ln x + \sqrt{1 - x^2}$$

$$\frac{x}{\sqrt{1 - x^2}} \ln x + \sqrt{1 - x^2} = \sqrt{1 - x^2} \cdot \frac{1}{x \sqrt{1 - x^2}} = 1 + \frac{x}{\sqrt{1 - x^2}}$$

$$\frac{x}{\sqrt{1 - x^2}} \ln x + \sqrt{1 - x^2} = 1$$

$$y = \cot^2 x + \arccos \sqrt{1 - x^2}$$

$$2 \cot x \csc^2 x = \frac{1}{\sqrt{1 - x^2}} = \frac{(x)}{\sqrt{1 - x^2}}$$

$$\frac{2 \cos x}{\sin^3 x} = \frac{x}{|x| \sqrt{1 - x^2}}$$

43

$$y = e^{x^2} \sin \sqrt{x} \quad y = \frac{\ln x + \cot x}{\sin x}$$

$$dy = \sin \sqrt{x} dx e^{x^2} + e^{x^2} d \sin \sqrt{x} = 2x e^{x^2} \sin \sqrt{x} dx + \frac{\cos \sqrt{x}}{2\sqrt{x}} e^{x^2} dx$$

$$e^{x^2} = 2x \sin \sqrt{x} + \frac{\cos \sqrt{x}}{2\sqrt{x}} dx$$

$$dy = \frac{\sin x d(\ln x + \cot x) + (\ln x + \cot x) d \sin x}{(\sin x)^2}$$

$$\frac{1}{\sin x} = \frac{1}{x} \csc^2 x dx + \cos x (\ln x + \cot x) dx$$

$$\frac{1}{x \sin x} \csc^3 x + \cos x \ln x + \cos x \cot x dx$$

$$44 \quad f(x) = x(x-1)(x-2) \cdots (x-100) \quad f(50)$$

$$g(x) = x(x-1)(x-2) \cdots (x-49)(x-50)$$

$$f(x) = (x - 50)g$$

$$f(x) = g(x) = (x - 50)$$

$$f(50) = g(50) = (50!)(-1)^{50}(50!) = (50!)^2$$

$$45 \quad f(x) = \int_y f(\ln x) e^{f(x)} dy$$

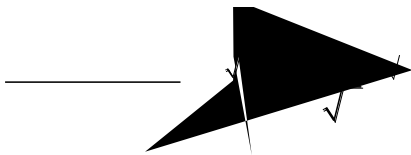
$$d_y (f(\ln x) e^{f(x)}) = f'(x) e^{f(x)} (1) = e^{f(x)}$$

$$f(x) e^{f(x)} f(\ln x) dx = \frac{1}{x} f(\ln x) e^{f(x)} dx$$

$$e^{f(x)} f(x) f(\ln x) = \frac{1}{x} f(\ln x) dx$$

$$46 \quad y = y(x) = \arctan(x^2 - y^2) = ye^{\sqrt{x}} \quad \frac{dy}{dx} = dy$$

$$\frac{1}{1 - x^2 - y^2} (2x - 2yy') = ye^{\sqrt{x}} = \frac{y}{2\sqrt{x}} e^{\sqrt{x}}$$



$$y = \frac{\frac{y}{2\sqrt{x}} e^{\sqrt{x}} - \frac{2x}{1 - x^2 - y^2}}{\frac{2y}{1 - x^2 - y^2} e^{\sqrt{x}}} = \frac{ye^{\sqrt{x}} - \frac{2x}{1 - x^2 - y^2}}{4y\sqrt{x} - \frac{2\sqrt{x}}{1 - x^2 - y^2} e^{\sqrt{x}}}$$

$$dy = \frac{ye^{\sqrt{x}} - \frac{2x}{1 - x^2 - y^2}}{4y\sqrt{x} - \frac{2\sqrt{x}}{1 - x^2 - y^2} e^{\sqrt{x}}} dx$$

$$d \arctan(x^2 - y^2) = d ye^{\sqrt{x}}$$

$$\frac{1}{1 - x^2 - y^2} d(x^2 - y^2) = e^{\sqrt{x}} dy = y de^{\sqrt{x}}$$

$$\frac{2}{1 - x^2 - y^2} x dx - y dy = e^{\sqrt{x}} dy = \frac{y}{2\sqrt{x}} e^{\sqrt{x}} dx$$

$$\frac{2y}{1 - x^2 - y^2} e^{\sqrt{x}} dy = \frac{y}{2\sqrt{x}} e^{\sqrt{x}} - \frac{2x}{1 - x^2 - y^2} dx$$

$$\frac{4y\sqrt{x} - 2\sqrt{x} - 1}{2\sqrt{x} - 1} \frac{x^2 - y^2}{y^2} e^{\sqrt{x}} dy = \frac{ye^{\sqrt{x}} - 1}{2\sqrt{x} - 1} \frac{x^2 - y^2}{y^2} 4\sqrt{x} dx$$

$$dy = \frac{ye^{\sqrt{x}} - 1}{4y\sqrt{x} - 2\sqrt{x} - 1} \frac{x^2 - y^2}{y^2} e^{\sqrt{x}} dx$$

$$\frac{\sqrt{y}}{\sqrt{x}} = \frac{\sqrt{y}}{\sqrt{x}}$$

$$47 \quad y = \sqrt[3]{\frac{x(x-1)(x-2)}{(x^2-1)(e^x-x)}} \quad y$$

$$\ln y = \frac{1}{3} \ln x - \ln(x-1) - \ln(x-2) + \ln(e^x-x)$$

x

$$\frac{1}{y} y = \frac{1}{3} \frac{1}{x} - \frac{1}{x-1} + \frac{1}{x-2} - \frac{2x}{x^2-1} + \frac{e^x}{e^x-x}$$

$$y = \frac{1}{3} \sqrt[3]{\frac{x(x-1)(x-2)}{(x^2-1)(e^x-x)}} \left(\frac{1}{x} - \frac{1}{x-1} + \frac{1}{x-2} - \frac{2x}{x^2-1} + \frac{e^x}{e^x-x} \right)$$

$$48 \quad \begin{aligned} x &= \ln(1+t^3) \\ y &= t^2 \sin t \end{aligned} \quad \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t \sin t + t^2 \cos t}{\frac{3t^2}{1+t^3}}$$

$$= \frac{(1+t^3)(2t \sin t + t^2 \cos t)}{3t^2} = \frac{(1+t^3)(2 \sin t + t \cos t)}{3t}$$

$$49 \quad y = \frac{1}{x}(x-0) = x_0, \frac{1}{x_0} \quad \text{四}$$

$$y = \frac{1}{x_0} - \frac{1}{x_0^2} x = x_0$$

$$y = 0 \quad X = 2x_0$$

$$x = 0 \quad Y = \frac{2}{x_0}$$

$$S = \frac{1}{2}XY = \frac{1}{2}(2x_0) \frac{2}{x_0} = 2$$

50
$$\begin{aligned} x &= 1+t^2 \\ y &= t^3 \end{aligned} \quad t=2$$

$$x_0 = 1, \quad y_0 = 2^3 = 8, \quad \frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3}{2}t$$

$$\left. \frac{dy}{dx} \right|_{t=2} = 3 \quad y-8 = 3(x-5)$$

$$3x - y - 7 = 0$$

51
$$\begin{aligned} x &= t^2 - 2t, \\ y &= \ln(1-t) \end{aligned} \quad y=y(x) \quad x=3 \quad x \quad \text{四}$$

(A) $\frac{1}{8} \ln 2 = 3$. (B) $\frac{1}{8} \ln 2 = 3$.

(C) $8 \ln 2 = 3$. (D) $8 \ln 2 = 3$. [A]

$$x=3, \quad t^2 - 2t = 3, \quad t = 1, t = 3 \quad y$$

$$\left. \frac{dy}{dx} \right|_{t=1} = \frac{1}{2t} \frac{1}{2} \bigg|_{t=1} = \frac{1}{8} \quad x=3(\quad y=\ln 2)$$

$$y = \ln 2 = 8(x-3)$$

$y=0, \quad x \quad \text{四} \quad \frac{1}{8} \ln 2 = 3, \quad (A).$

52
$$\begin{aligned} x &= t^3 - 3t + 1 \\ y &= t^3 - 3t + 1 \end{aligned} \quad y = y(x) \quad x$$

$$x = x(t)$$

$$y = y(t)$$

$$\frac{d^2 y}{dx^2} = \frac{y(t)x(t) - x(t)y(t)}{(x(t))^3} \quad \text{内} \quad \frac{d^2 y}{dx^2} = 0 \quad x$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{3t^2}{3t^2} - \frac{3}{3}}{\frac{t^2}{t^2} - \frac{1}{1}} = 1 - \frac{2}{t^2 - 1},$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \frac{dy}{dx} \frac{dt}{dx} = 1 - \frac{2}{t^2 - 1} - \frac{1}{3(t^2 - 1)} - \frac{4}{3t^2},$$

$$\frac{d^2 y}{dx^2} = 0 \quad t = 0.$$

$$x = t^3 - 3t - 1, \quad t = 0, \quad x \in (-1, 1) \quad (\because t = 0 \Rightarrow x = -1 \Rightarrow x \in (-1, 1]) \quad .)$$

$$53 \quad f(x) = 0 \quad 3 \quad 0 \quad 3 \quad f(0) \quad f(1) \quad f(2) \quad 3 \quad f(3) \quad 1$$

$$0 \quad 3 \quad f \quad 0$$

$$\because f(x) = 0 \quad 3 \quad f(x) = 0 \quad 2 \quad M \quad m \quad m \quad f(0) \quad M$$

$$m \quad f(1) \quad M \quad m \quad f(2) \quad M \quad m \quad \frac{1}{3} \quad f(0) \quad f(1) \quad f(2) \quad M$$

$$c = 0 \quad 2$$

$$f(c) = \frac{1}{3} \quad f(0) \quad f(1) \quad f(2) \quad 1$$

$$f(c) = f(3) \quad f(x) = c \quad 3 \quad c \quad 3 \quad \text{内} \quad c \quad 3 \quad 0 \quad 3$$

$$f = 0$$

$$54 \quad f(x) = 0 \quad 1 \quad 0 \quad 1 \quad 3 \quad \frac{1}{3} \quad f(x) \, dx \quad f(0)$$

$$x \in (-1, 1) \quad f(x) = 0$$

$$\frac{1}{3} \quad f(x) \, dx \quad f(c) = 1 \quad \frac{2}{3}$$

$$f(c) = 3 \quad \frac{1}{3} \quad f(x) \, dx \quad ($$

$$f(x) = 0 \quad c$$

$$x \in (0, c) \quad (0, 1) \quad f(x) = 0$$

$$55 \quad x > 0 \quad \frac{x}{1+x} < \ln(1+x) < x$$

$$f(t) = \ln(1+t) \quad [0, x]$$

$$f(t) = \frac{1}{1+t} \quad \ln(1+x) - \ln 1 = \frac{1}{1+x} [x - 0] \quad (0 < x < x)$$

$$\ln \left(\frac{1}{1+x} \right) = -\frac{x}{1+x} \quad (0 < x < x)$$

$$\frac{x}{1+x} < \ln \left(\frac{1}{1+x} \right) < x$$

$$56 \quad f(x) = a \quad b \quad a \quad b \quad f(a) \quad f(b) \quad a \quad b$$

$$f = 0$$

$$c(a, b) = f(c) - f(a) - f(b)$$

$$\begin{aligned} \text{复 } f(c) - f(a) - f(x) &= a - c & x_1 &= (a, c) & f_1 &= \frac{f(c) - f(a)}{c - a} & 0 \\ \text{复 } f(b) - f(c) - f(x) &= c - b & x_2 &= (c, b) & f_2 &= \frac{f(b) - f(c)}{b - c} & 0 \end{aligned}$$

$$x(a, b) = f = 0$$

$$57 \quad f(x) = 0 \quad f(0) = 0 \quad \text{了 } x_1 > 0 \quad x_2 > 0$$

$$f(x_1 + x_2) < f(x_1) + f(x_2)$$

$$x_1 = x_2$$

$$f(x_1) = f(x_1) - f(0) = (x_1 - 0)f(x_1) \quad 0 < x_1 < x_1$$

$$f(x_1 + x_2) - f(x_2) = [(x_1 + x_2) - x_2]f(x_2) \quad x_2 < x_2 < x_1 + x_2 \quad x_1 < x_2$$

$$f(x) < 0 \quad f(x) \quad f(x_1) > f(x_2)$$

$$f(x_1) > f(x_1 + x_2) - f(x_2)$$

$$f(x_1 + x_2) < f(x_1) + f(x_2)$$

$$58 \quad f(x) = a - b \quad a - b \quad b - a = 0 \quad x(a, b) = h(a, b)$$

$$f(h) = \frac{a+b}{2} g \frac{f(x)}{x}$$

$$g(x)$$

$$\frac{f(x)}{g(x)} = \frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f(h)(b - a)}{g(b) - g(a)}$$

$$\frac{f(x)}{2x} = \frac{f(h)}{a+b} = \frac{f(h)(b-a)}{b^2 - a^2}$$

$$\text{内 } g(x) = x^2$$

$$f(h) = \frac{b^2 + ab + a^2}{3} g \frac{f(x)}{x^2} \quad g(x) = x^3$$

$$59 \quad f(x) = 01 \quad f(0) = f(0) = f(1) = 0 \quad f(1) = 1$$

$$x \in (0, 1) \quad |f(x)| \leq 4$$

$$f(x) = x \quad x=0 \quad \text{令}$$

$$f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \dots \quad (0 \leq x \leq 1)$$

$$f(x) = x \quad x=1 \quad \text{令}$$

$$f(x) = f(1) + f'(1)(x-1) + \frac{1}{2!}f''(1)(x-1)^2 + \dots \quad (x \in [0, 1])$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{8}f''\left(\frac{1}{2}\right) \quad \left(0 \leq \frac{1}{2} \leq 1\right)$$

$$f\left(\frac{1}{2}\right) = \frac{1}{8}f''\left(\frac{1}{2}\right) \quad \left(\frac{1}{2} \leq 1\right)$$

$$f(x_1) + f(x_2) \leq 8 \quad |f(x_1)| + |f(x_2)| \leq 8$$

$$\max\{|f(x_1)|, |f(x_2)|\} \leq 4$$

$$x \in (0, 1) \quad |f(x)| \leq 4$$

$$60 \quad 0,1 \quad f(x) = 0 \quad f(0) = f(1) = f(0) = f(0) = f(1)$$

$$A \quad f(1) = f(0) = f(1) = f(0) \quad B \quad f(1) = f(1) = f(0) = f(0)$$

$$C \quad f(1) = f(0) = f(1) = f(0) \quad D \quad f(1) = f(0) = f(1) = f(0)$$

B

$$f(1) = f(0) = f(1) = 0 = f(0)$$

$$0 \leq 1 \quad f(x) = 0 \quad f(x) \quad \text{别}$$

$$f(1) = f(0) = f(1)$$

$$61 \quad f(x) = a, b \quad a, b \quad f(a) = 0 \quad \text{复} \quad f(x) \quad \text{别} \quad (x) = \frac{f(x)}{x-a}$$

a, b 别

$$(x) = \frac{x-a}{x-a^2} \quad \frac{f(x)}{x-a^2}$$

$$f(x) = f(x) = f(a) = f(x-a) \quad (a \leq x)$$

$$(x) \frac{f(x) - f(a)}{x - a}$$

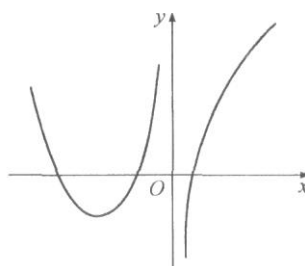
$f(x)$ 别 $f(x)$ f
 $x = 0$ $x = a, b$ 别

62 $f(x) = (\quad , \quad)$

复 $f(x)$

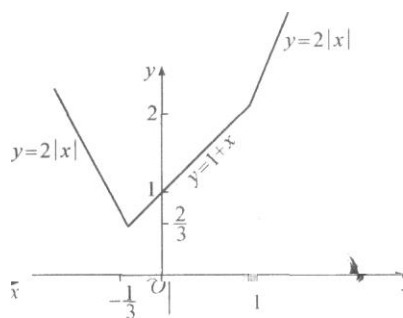
A
B
C

D



63 $f(x) = \max\{2|x|, |1-x|\}$

$$f(x) = \begin{cases} 1-x & -\frac{1}{3} \leq x \leq 1 \\ 2|x| & x < -\frac{1}{3} \text{ or } x > 1 \end{cases}$$



$$f \quad \frac{1}{3} = \frac{2}{3}$$

$$64 \quad f(x) \quad x_0$$

$$\lim_{n \rightarrow \infty} \frac{f(x) - f(x_0)}{(x - x_0)^n} = k \quad n \quad k = 0 \quad n \quad f(x_0)$$

$$\lim_{x \rightarrow x_0} a(x) = 0$$

$$f(x) - f(x_0) = k(x - x_0)^n + a(x)(x - x_0)^n$$

$$n \quad |x - x_0| < d$$

$$f(x) - f(x_0) = k \quad k = 0 \quad f(x_0) \quad k = 0 \quad f(x_0)$$

$$n \quad |x - x_0| < d \quad f(x) - f(x_0) \quad x_0 \quad f(x_0)$$

$$65 \quad f(x) = \int_0^1 |t - x| dt = 0 \quad x = 1 \quad f(x)$$

$$f(x) = \int_0^x t(x-t) dt + \int_x^1 t(t-x) dt = \int_0^x (tx - t^2) dt + \int_x^1 (t^2 - tx) dt$$

$$\left(x \frac{t^2}{2} - \frac{t^3}{3} \right) \Big|_0^x - \left(\frac{t^3}{3} - x \frac{t^2}{2} \right) \Big|_x^1 = \left(\frac{x^3}{2} - \frac{x^3}{3} \right) - \left(\frac{1}{3} - \frac{x}{2} \right) = \left(\frac{x^3}{3} - \frac{x^3}{2} \right)$$

$$\frac{x^3}{6} - \frac{1}{3} \frac{x}{2} = \frac{x^3}{6} - \frac{x^3}{3} = \frac{x}{2} - \frac{1}{3}.$$

$$f(x) = x^2 - \frac{1}{2}, \quad f(x) = 0 \quad x = \frac{\sqrt{2}}{2}, \quad 0 \leq x \leq 1 \quad x = \frac{\sqrt{2}}{2}.$$

$$f(x) = 0 \quad \frac{\sqrt{2}}{2} \leq x \leq 1$$

$$f(x) = 0 \quad 0 \leq x \leq \frac{\sqrt{2}}{2}$$

$$f(x) \quad \left(\frac{\sqrt{2}}{2}, 1 \right) \quad \left(0, \frac{\sqrt{2}}{2} \right).$$

$$f(x) = 2x \quad (0, 1).$$

$$f\left(\frac{\sqrt{2}}{2}\right) = 0, f\left(\frac{\sqrt{2}}{2}\right) = 0, f\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{6} - \frac{1}{3}.$$

$$y = e^{x \ln(\sin x)} [\ln(\sin x) + x \frac{\cos x}{1 - \sin x}]$$

$$dy \Big|_x = y (\quad) dx \quad dx.$$

$$\ln y = x \ln(1 - \sin x) - x$$

$$\frac{1}{y} \ln(1 - \sin x) = \frac{x \cos x}{1 - \sin x}$$

$$y = (1 - \sin x)^x \left[\ln(1 - \sin x) + x \frac{\cos x}{1 - \sin x} \right]$$

$$dy \Big|_x = y (\quad) dx \quad dx.$$

67 $y = \frac{(1-x)^{\frac{3}{2}}}{\sqrt{x}}$ _____

$$a = \lim_{x \rightarrow 1} \frac{f(x)}{x} = \lim_{x \rightarrow 1} \frac{(1-x)^{\frac{3}{2}}}{x\sqrt{x}} = 1,$$

$$b \lim_{x \rightarrow 0} f(x) = ax \lim_{x \rightarrow 0} \frac{(1-x)^{\frac{3}{2}} x^{\frac{3}{2}}}{\sqrt{x}} = \frac{3}{2}$$

$$y = x + \frac{3}{2}.$$

复

68. $\lim_{x \rightarrow 0} \frac{f(x)}{x} = a$ where $f(x) = kx^2 \sqrt{1 - x \arcsin x} - \sqrt{\cos x}$. Find k .

$$\lim_{x \rightarrow 0} \frac{(x)^k}{(x)} = 1 \quad k > 1.$$

$$\lim_{x \rightarrow 0} \frac{(x)}{(x)} \quad \lim_{x \rightarrow 0} \frac{\sqrt{1-x} \arcsin x}{kx^2} \quad \sqrt{\cos x}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{x \arcsin x - \frac{1}{2} \cos x}{kx^2 (\sqrt{1-x^2} \arcsin x - \sqrt{\cos x})} \\
 &= \frac{1}{2k} \lim_{x \rightarrow 0} \frac{x \arcsin x - \frac{1}{2} \cos x}{x^2} = \frac{3}{4k} - \frac{1}{k} = \frac{3}{4}.
 \end{aligned}$$

务

69 $f(x) = \lim_{n \rightarrow \infty} \sqrt[n]{1 + |x|^{3n}}$ $f(x) = (\quad , \quad)$

(A)

(B)

(C)

(D)

[]

内 $f(x)$

$$|x| \leq 1 \implies f(x) = \lim_{n \rightarrow \infty} \sqrt[n]{1 - |x|^{3n}} = 1$$

$$|x| > 1 \quad f(x) = \lim_{n \rightarrow \infty} \sqrt[n]{1 - \frac{1}{n}}$$

$$|x| > 1 \quad f(x) = \lim_{n \rightarrow \infty} |x|^3 \left(\frac{1}{|x|^{3n}} - 1 \right)^{\frac{1}{n}} = |x|^3.$$

$$f(x) = \begin{cases} x^3, & x \leq 1, \\ 1, & 1 < x < 1, \\ x^3, & x \geq 1. \end{cases} \quad f(x) \text{ 在 } x=1 \text{ 处连续} \quad (C).$$

70
$$f(x) = \frac{1}{e^{\frac{x}{x-1}} - 1}$$

(A) $x=0, x=1$ $f(x)$ 连续.

B $x=0, x=1$ $f(x)$ 不连续.

(C) $x=0$ $f(x)$ 连续, $x=1$ $f(x)$ 不连续.

(D) $x=0$ $f(x)$ 不连续, $x=1$ $f(x)$ 连续. []

$x=0, x=1$ $f(x)$ 连续.

$f(x)$ $x=0, x=1$ 连续.

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f(x)$$

$$\lim_{x \rightarrow 1} f(x) = 0, \lim_{x \rightarrow 1} f(x) = 1 \quad x=1 \quad (D).$$

$$\lim_{x \rightarrow 1} \frac{x}{x-1} = \lim_{x \rightarrow 1} \frac{x}{x-1} = \lim_{x \rightarrow 1} e^{\frac{x}{x-1}} = \lim_{x \rightarrow 1} e^{\frac{x}{x-1}} = 0.$$

71 $x > 0 \quad (1 - ax^2)^{\frac{1}{4}} - 1 \sim x \sin x$

a = _____.

$$\lim_{x \rightarrow 0} \frac{(1 - ax^2)^{\frac{1}{4}} - 1}{x \sin x} = 1$$

$$y=f(x) \quad x=0 \quad n \quad f^{(n)}(0) \quad x^n$$

$$y = 2^x \ln 2 \quad \dots, y^{(x)} = 2^x (\ln 2)^n$$

$$y^{(n)}(0) = (\ln 2)^n \quad x^n \quad \frac{y^{(n)}(0)}{n!} = \frac{(\ln 2)^n}{n!}.$$

$$74 \quad \{a_n\}, \{b_n\}, \{c_n\} \quad \lim_n a_n = 0, \lim_n b_n = 1, \lim_n c_n = \quad ,$$

$$(A) \quad a_n b_n \rightarrow n \quad . \quad (B) \quad b_n c_n \rightarrow n \quad .$$

$$(C) \quad \lim_n a_n c_n \quad . \quad (D) \quad \lim_n b_n c_n \quad . \quad [\quad]$$

$$(A), (B) \quad \lim_n a_n c_n = 0$$

$$\lim_n b_n c_n = 1 \quad .$$

$$a_n = \frac{2}{n} \quad b_n = 1$$

$$= 4 \lim_{x \rightarrow 0} \frac{e^{ax} x^2}{x^2} \frac{ax}{1} = 4 \lim_{x \rightarrow 0} \frac{ae^{ax} 2x}{2x} = 2a^2 \quad 4.$$

$$f(0) = 0, \quad f'(0) = 6a - 2a^2 - 4 = a - 1 - a = -2.$$

$$a = -1, \quad \lim_{x \rightarrow 0} f(x) = 6 - f(0) = f(x) \quad x=0.$$

$$a = -2, \quad \lim_{x \rightarrow 0} f(x) = 12 - f(0) = f(x) \quad x=0.$$

务.

$$76 \quad y = y(x) \quad x = 1 - 2t^2, \quad y = \frac{1}{1 - 2\ln t} e^u \frac{du}{u} (t = 1) \quad \frac{d^2 y}{dx^2} \Big|_{x=9}.$$

x=9

t

$$\frac{dy}{dt} = \frac{e^{1-2\ln t}}{1-2\ln t} \cdot \frac{2}{t} = \frac{2et}{1-2\ln t} \quad \frac{dx}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2et}{1-2\ln t}}{4t} = \frac{e}{2(1-2\ln t)},$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{1}{\frac{dx}{dt}} = \frac{e}{2} \frac{1}{(1-2\ln t)^2} \cdot \frac{2}{t} = \frac{1}{4t}$$

$$= \frac{e}{4t^2(1-2\ln t)^2}.$$

$$x=9 \quad x = 1 - 2t^2 \text{ 单 } t > 1 \quad t=2,$$

$$\frac{d^2 y}{dx^2} \Big|_{x=9} = \frac{e}{4t^2(1-2\ln t)^2} \Big|_{t=2} = \frac{e}{16(1-2\ln 2)^2}.$$

$$77 \quad f(x) = \lim_n \frac{(n-1)x}{nx^2-1}, \quad f(x) = x \quad \text{---}.$$

x

内 $f(x)$

,

$f(x)$

$$x = 0 \quad f(x) = 0$$

$$x = 0 \quad f(x) = \lim_n \frac{(n-1)x}{nx^2-1} = \lim_n \frac{(1-\frac{1}{n})x}{x^2-\frac{1}{n}} = \frac{x}{x^2} = \frac{1}{x}$$

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{x}, & x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1}{x} \neq f(0)$$

$$x \rightarrow 0 \quad f(x) \quad .$$

78 $y(x) = \begin{cases} x - t^3 - 3t - 1, & t \geq 0 \\ y = y(x) \end{cases}$, $y = y(x)$ x

$$x = x(t)$$

$$y = y(t)$$

$$\frac{d^2 y}{dx^2} = \frac{y'(t)x'(t) - x(t)y''(t)}{(x'(t))^3} \quad \text{内} \quad \frac{d^2 y}{dx^2} = 0 \quad x \quad .$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{3t^2 - 3} = \frac{t^2 - 1}{t^2 - 1} = 1 - \frac{2}{t^2 - 1},$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx} = 1 - \frac{2}{t^2 - 1} = \frac{1}{3(t^2 - 1)} - \frac{4}{3t^3},$$

$$\frac{d^2 y}{dx^2} = 0 \quad t = 0.$$

$$x = t^3 - 3t - 1, \quad t = 0, \quad x \in (-1, 1) \quad (\because t = 0 \quad x = -1 \quad x \in (-1, 1]) \quad .)$$

79 $x = 0$ $\int_0^x \cos t^2 dt$, $\int_0^{x^2} \tan \sqrt{t} dt$, $\int_0^{\sqrt{x}} \sin t^3 dt$,

,

A , , .

B , , .

C , , .

D , , .

域

$$\therefore \lim_{x \rightarrow 0} \frac{\int_0^{\sqrt{x}} \sin t^3 dt}{\int_0^x \cos t^2 dt}$$

$$\lim_{x \rightarrow 0} \frac{\sin x^{\frac{3}{2}}}{\cos x^2} = \frac{1}{2\sqrt{x}}$$

$$\lim_{x \rightarrow 0} \frac{x^{\frac{3}{2}}}{2\sqrt{x}} = \lim_{x \rightarrow 0} \frac{x}{2} = 0$$

o()

$$\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \tan \sqrt{t} dt}{\int_0^{\sqrt{x}} \sin t^3 dt} = \lim_{x \rightarrow 0} \frac{\tan x \cdot 2x}{\sin x^{\frac{3}{2}} \cdot \frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow 0} \frac{2x^2}{\frac{1}{2}x} = 0$$

o()

, B .

80 $f(x) = |x(1-x)|$,A $x=0$ 是 $f(x)$ 的极值点, $(0,0)$ 是 $y=f(x)$ 的拐点.B $x=0$ 不是 $f(x)$ 的极值点, $(0,0)$ 是 $y=f(x)$ 的拐点.C $x=0$ 是 $f(x)$ 的极值点, $(0,0)$ 不是 $y=f(x)$ 的拐点.D $x=0$ 不是 $f(x)$ 的极值点, $(0,0)$ 不是 $y=f(x)$ 的拐点. $x=0$ 是 $f(x)$ 的极值点, $f'(x)$ 在 $x=0$ 处不可导.

$$f(x) = \begin{cases} x(1-x), & 1-x \leq 0 \\ x(1-x), & 0 \leq x \leq 1 \end{cases}$$

$$f'(x) = \begin{cases} 1-2x, & 1-x \leq 0 \\ 1-2x, & 0 \leq x \leq 1 \end{cases}$$

$$f''(x) = \begin{cases} -2, & 1-x \leq 0 \\ -2, & 0 \leq x \leq 1 \end{cases}$$

 $1-x \leq 0$ 时, $f'(x) = 1-2x$, $1-x \leq 0$ 时, $f'(x) = 1-2x$, $(0,0)$ 是拐点. $f(0) = 0$, $x=0$ 是 $f(x)$ 的极值点, $x=0$ 不是 $y=f(x)$ 的拐点., $x=0$ 不是 $f(x)$ 的极值点, $(0,0)$ 是 $y=f(x)$ 的拐点, C .81 $f(x)$ 在 $x=0$ 处可导, $f'(0) = 0$, $f''(0) = 0$,A $f(x)$ 在 $(0, +\infty)$ 内单调递增.B $f(x)$ 在 $(-\infty, 0)$ 内单调递减.C 了 $x \in (0, +\infty)$ 时, $f(x) > f(0)$.D 了 $x \in (-\infty, 0)$ 时, $f(x) > f(0)$. ()

单

 $f(x) = x^2$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = 0,$$

$$, \quad 0, \quad |x|, \quad ,$$

$$\frac{f(x) - f(0)}{x} = 0$$

$$x \rightarrow 0, \quad f(x) = f(0)$$

$$x \rightarrow 0, \quad f(x) = f(0),$$

C .

$$82 \quad \lim_{x \rightarrow 0} \frac{1}{x^3} = \frac{2 - \cos x}{3} = 1.$$

$$\frac{0}{0} \quad \text{域}$$

$$\lim_{x \rightarrow 0} \frac{e^{x \ln \frac{2 - \cos x}{3}} - 1}{x^3}$$

$$\lim_{x \rightarrow 0} \frac{\ln \frac{2 - \cos x}{3}}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\ln \frac{2 - \cos x}{3}}{x^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{1}{2 - \cos x} = \frac{1}{2}$$

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{2 - \cos x} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\lim_{x \rightarrow 0} \frac{e^{x \ln \frac{2 - \cos x}{3}} - 1}{x^3}$$

$$\lim_{x \rightarrow 0} \frac{\ln \frac{2 - \cos x}{3}}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\ln \frac{2 - \cos x}{3}}{x^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x}{3x^2} = \frac{1}{6}$$

$$83 \quad f(x) = x(x^2 - 4), \quad [0, 2], \quad f(x) = x(x^2 - 4), \quad \text{了 } x$$

$$f(x) = k f(x - 2), \quad k = \frac{1}{2}.$$

$$2x^2 - 0 = 0 \quad x^2 - 2 = 2$$

$$f(x) = k f(x-2) = k(x-2)[(x-2)^2 - 4] = kx(x-2)(x-4).$$

$$f(0) = 0.$$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{x(x^2 - 4)}{x} = -4$$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{kx(x-2)(x-4)}{x} = -8k.$$

$$f'(0) = f'(0), \quad k = \frac{1}{2}.$$

$$k = \frac{1}{2}, \quad f(x) = x^2 - 2.$$

$$84 \quad e^a - b = e^2, \quad \ln^2 b - \ln^2 a = \frac{4}{e^2}(b - a).$$

$$(x) = \ln^2 x - \frac{4}{e^2}x,$$

$$(x) = 2\frac{\ln x}{x} - \frac{4}{e^2}$$

$$(x) = \frac{1}{2} \frac{1 - \ln x}{x^2},$$

$$x = e, \quad (x) = 0, \quad (x) = \frac{4}{e^2}, \quad e = x = e^2,$$

$$(x) = (e^2) = \frac{4}{e^2} - \frac{4}{e^2} = 0,$$

$$e = x = e^2, \quad (x) = \text{别}.$$

$$, \quad e^a - b = e^2, \quad (b) = (a),$$

$$\ln^2 b - \frac{4}{e^2}b = \ln^2 a - \frac{4}{e^2}a$$

$$\ln^2 b - \ln^2 a = \frac{4}{e^2}(b - a).$$

$$(x) = \ln^2 x - \ln^2 a - \frac{4}{e^2}(x - a),$$

$$(x) = 2\frac{\ln x}{x} - \frac{4}{e^2}$$

$$(x) = \frac{1}{2} \frac{1 - \ln x}{x^2},$$

$$x = e, \quad (x) = 0 \quad (x) = \frac{4}{e^2}, \quad e = x = e^2,$$

$$(x) \quad (e^2) \quad \frac{4}{e^2} \quad \frac{4}{e^2} \quad 0,$$

$$e \quad x \quad e^2, \quad (x) \quad \text{别}.$$

$$e \quad a \quad b \quad e^2, \quad (x) \quad (a) \quad 0 \quad x \quad b \quad (b) \quad 0$$

$$\ln^2 b \quad \ln^2 a \quad \frac{4}{e^2}(b \quad a).$$

$$\ln^2 x \quad [a, b] \quad ,$$

$$\ln^2 b \quad \ln^2 a \quad \frac{2\ln}{(b \quad a)}, \quad a \quad b.$$

$$(t) \quad \frac{\ln t}{t}, \quad (t) \quad \frac{1 \quad \ln t}{t^2},$$

$$t \quad e, \quad (t) \quad 0, \quad (t) \quad ,$$

$$(\quad) \quad (e^2),$$

$$\frac{\ln}{e^2} \quad \frac{\ln e^2}{e^2} \quad \frac{2}{e^2},$$

$$\ln^2 b \quad \ln^2 a \quad \frac{4}{e^2}(b \quad a)$$

$$85 \quad y \quad \frac{x \quad 4\sin x}{5x \quad 2\cos x} \quad \underline{\hspace{2cm}}$$

$$\therefore \lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} \frac{1 \quad \frac{4\sin x}{x}}{5 \quad \frac{2\cos x}{x}} = \frac{1}{5}$$

$$86 \quad f(x) = \frac{1}{x^3} \int_0^x \sin t^2 dt, \quad x > 0 \quad x=0 \quad a = \underline{\hspace{2cm}}$$

$$a, \quad x < 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{sm(x^2)}{3x^2} = \frac{1}{3}$$

$$87 \quad y = y(x) \quad y = 1 \quad xe^y \quad \left. \frac{dy}{dx} \right|_{x=0} \quad \underline{\hspace{2cm}}$$

$$x=0 \quad y=1$$

$$x \quad y \quad e^y \quad xe^y y$$

$$y(1 - xe^y) = e^y \quad y \Big|_{x=0} = \frac{e^y}{1 - xe^y} \Big|_{x=0, y=1} = e$$

$$88 \quad y = f(x) \quad f(x) = 0, \quad f'(x) = 0, \quad x = x_0$$

$$y \, dy = f(x) \, dx, \quad x = 0$$

$$A \quad 0 \, dy = y \quad B \quad 0 \, y \, dy$$

$$C \quad y \, dy = 0 \quad D \quad dy = y = 0$$

$$f(x) = 0 \quad f(x) \quad \text{别}$$

$$f(x) = 0 \quad f(x)$$

$$89 \quad g(x) = e^{1-g(x)}, h(1) = 1, g(1) = 2, \quad g(1)$$

$$A \quad \ln 3 = 1 \quad B \quad \ln 3 = 1$$

$$C \quad \ln 2 = 1 \quad D \quad \ln 2 = 1$$

$$h(x) = g(x)e^{1-g(x)} = 1 - 2e^{1-g(1)}$$

$$90 \quad A, B, C \quad e^x(1 - Bx - Cx^2) = 1 - Ax - o(x^3) \quad o(x^3) \quad x = 0 \quad x^3$$

$$1 - \frac{2}{3} - \frac{3}{4} = \left(\frac{3}{4}\right)$$

$$\left[1 - x - \frac{x^2}{2} - \frac{x^3}{6} - o(x^3)\right] \left[1 - Bx - Cx^2\right] = 1 - Ax - o(x^3)$$

$$1 - (B-1)x - (C-B)\frac{1}{2}x^2 - \frac{B}{2} - C\frac{1}{6} - o(x^3) = 1 - Ax - o(x^3)$$

$$B+1=A$$

$$C+B+\frac{1}{2}=0$$

$$\frac{B}{2} - C - \frac{1}{6} = 0$$

$$- \frac{B}{2} - \frac{1}{3} = 0 \quad B = \frac{2}{3}$$

$$A = \frac{1}{3}$$

$$C = \frac{1}{6}$$

$$91 \quad \{x_n\} \quad 0 \quad x_1 \quad x_{n-1} \quad \sin x_n (n=1, 2, 3, \dots)$$

$$1 \quad \lim_{n \rightarrow \infty} x_{n-1}$$

$$2 \quad \lim_{n \rightarrow \infty} \frac{x_{n-1}}{x_n^{\frac{1}{2}}}$$

$$1 \quad \because x_2 = \sin x_1, \quad 0 < x_2 < 1, \quad n \geq 2$$

$$x_{n-1} = \sin x_n \in x_n, \{x_n\} \quad \because x_n > 0$$

$$1 \quad \lim_{n \rightarrow \infty} x_n = A$$

$$x_{n-1} = \sin x_n \quad A = \sin A \quad A = 0$$

$$\lim_{n \rightarrow \infty} x_{n-1} = 0$$

$$2 \quad \lim_{n \rightarrow \infty} \frac{\sin x_n}{x_n^{\frac{1}{2}}} = "1"$$

$\therefore$

$$\lim_{t \rightarrow 0} \frac{\sin t}{t} = e^{\lim_{t \rightarrow 0} \frac{1}{t^2} \cdot \frac{1}{t} \cdot \frac{\sin t}{t}}$$

$$e^{\lim_{t \rightarrow 0} \frac{1}{2t} \cdot \frac{1}{\frac{\sin t}{t}} \cdot \frac{(t \cos t - \sin t)}{t^2}}$$

$$e^{\lim_{t \rightarrow 0} \frac{t \cos t - \sin t}{2t^3}} = e^{\lim_{t \rightarrow 0} \frac{t(1 - \frac{t^2}{2} - 0(t^2)) - t(\frac{t^3}{6} - 0(t^3))}{2t^3}}$$

$$e^{\lim_{t \rightarrow 0} \frac{\frac{1}{2} - \frac{1}{6}t^2 - 0(t^3)}{2t^3}} = e^{\frac{1}{6}}$$

$$92 \quad 0 < a < b \quad b \sin b - 2 \cos b > b - a \sin a - 2 \cos a > \frac{1}{a}$$

$$f(x) = x \sin x - 2 \cos x \quad x$$

$$0 < a < x \quad , \quad f(x) \quad \text{别}$$

$$f(x) = \sin x - x \cos x - 2 \sin x$$

$$x \cos x - \sin x$$

$$f(x) = \cos x - x \sin x - \cos x - x \sin x = 0$$

$$f(x)$$

$$f(x) = \cos x = 0$$

$$0 < a < x \quad f(x) > 0 \quad f(x) \quad \text{别}$$

$$b \quad a \quad f(b) \quad f(a)$$

$$93 \quad L \quad \begin{matrix} x & t^2 & 1 \\ y & 4t & t^2 \end{matrix} \quad (t \geq 0)$$

$$I \quad L$$

$$II \quad (1, 0) \quad L \quad (x_0, y_0) \quad \text{内}$$

$$III \quad L \quad x \quad x_0 \quad \text{单 } x$$

$$I \quad \frac{dx}{dt} = 2t, \frac{dy}{dt} = 4 - 2t, \frac{dy}{dx} = \frac{4 - 2t}{2t} = \frac{2}{t} - 1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \frac{\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{1}{\frac{dx}{dt}} \left(\frac{2}{t^2} - \frac{1}{2t} \right) = \frac{1}{t^3} = 0 \quad (t \geq 0)$$

$$L(t \geq 0)$$

$$II \quad y = 0 \quad \frac{2}{t} = 1 \quad (x = 1) \quad x_0 = t_0^2 = 1 \quad y_0 = 4t_0 = t_0^2$$

$$4t_0 = t_0^2 \quad \frac{2}{t_0} = 1 \quad (t_0^2 = 2), 4t_0^2 = t_0^3 \quad (2 = t_0)(t_0^2 = 2)$$

$$t_0^2 = t_0 = 2 = 0, (t_0 = 1)(t_0 = 2) = 0 \because t_0 = 0 \quad t_0 = 1$$

$$2 = 3 \quad y = x = 1$$

$$III \quad L \quad x \quad g(y)$$

$$S \int_0^3 g(y) (y - 1) dy$$

$$t^2 = 4t \quad y = 0 \quad \text{内 } t = 2 \quad \sqrt{4 - y} = x = 2 \quad \sqrt{4 - y}^2 = 1$$

$$2 = 3 \quad L \quad y = 3 \quad x = 2 \quad x = 2 \quad \sqrt{4 - y}^2 = 1 \quad g(y)$$

$$S \int_0^3 (9 - y - 4\sqrt{4 - y}) (y - 1) dy$$

$$\int_0^3 (10 - 2y) dy - 4 \int_0^3 \sqrt{4 - y} dy$$

$$(10y - y^2) \Big|_0^3 - 4 \int_0^3 \sqrt{4 - y} d(4 - y) = 21 - 4 \left[\frac{2}{3} (4 - y)^{\frac{3}{2}} \right]_0^3$$

$$21 - \frac{8}{3} - \frac{64}{3} = 3 - \frac{2}{3}$$

$$94 \quad x = 0 \quad \sqrt{x}$$

(A) $1 - e^{\sqrt{x}}$. (B) $\ln(1 - \sqrt{x})$. (C) $\sqrt{1 - \sqrt{x}} - 1$. (D) $1 - \cos \sqrt{x}$.

(B).

务

内

$$\begin{aligned} x \rightarrow 0 \quad 1 - e^{\sqrt{x}} & \quad (e^{\sqrt{x}} - 1) \sim \sqrt{x} \quad \ln(1 - \sqrt{x}) \sim -\sqrt{x} \\ \sqrt{1 - \sqrt{x}} - 1 & \sim -\frac{1}{2}\sqrt{x} \quad 1 - \cos \sqrt{x} \sim \frac{1}{2}(\sqrt{x})^2 = \frac{1}{2}x. \end{aligned} \quad (B).$$

95 $f(x) \rightarrow 0$ 合

(A) $\lim_{x \rightarrow 0} \frac{f(x)}{x} \quad f(0)=0.$ (B) $\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{x} \quad f(0)=0.$

(C) $\lim_{x \rightarrow 0} \frac{f(x)}{x} \quad f(0) \neq 0.$ (D) $\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{x} \quad f(0) \neq 0.$

(D).

(A),(B) $\lim_{x \rightarrow 0} \frac{f(x)}{x} = \frac{f(0) - f(0)}{0 - 0} = \frac{0}{0}$ 内 $f(0)=0.$

$f(x) = |x| \quad x=0$

$\lim_{x \rightarrow 0} \frac{f(x) - f(-x)}{x} = \lim_{x \rightarrow 0} \frac{|x| - |-x|}{x} = 0 \quad f(x) = |x| \quad x=0$

96 $y = \frac{1}{x} \ln(1 - e^x)$

(A) 0. (B) 1. (C) 2. (D) 3.

(D).

内

$\lim_{x \rightarrow 0} \left[\frac{1}{x} \ln(1 - e^x) \right] \quad x \rightarrow 0$

$\lim_{x \rightarrow 0} \left[\frac{1}{x} \ln(1 - e^x) \right] = 0 \quad y=0$

$\lim_{x \rightarrow 0} \frac{y}{x} = \lim_{x \rightarrow 0} \left[\frac{1}{x^2} \frac{\ln(1 - e^x)}{x} \right] = \lim_{x \rightarrow 0} \frac{\ln(1 - e^x)}{x} = \lim_{x \rightarrow 0} \frac{e^x}{1 - e^x} = 1$

$\lim_{x \rightarrow 0} \left[\frac{1}{x} \ln(1 - e^x) \right] = \lim_{x \rightarrow 0} \left[\ln(1 - e^x) \right] = \lim_{x \rightarrow 0} [\ln(1 - e^x) - x]$

$= \lim_{x \rightarrow 0} [\ln e^x (1 - e^{-x}) - x] = \lim_{x \rightarrow 0} \ln(1 - e^{-x}) = 0$

$y = x.$ (D).

97 $\lim_{x \rightarrow 0} \frac{x^3 - x^2 - 1}{2^x - x^3} (\sin x - \cos x) = \underline{\hspace{2cm}}$

0.

$\lim_{x \rightarrow 0} \frac{x^3 - x^2 - 1}{2^x - x^3} = 0 \quad \sin x + \cos x$

$$\lim_{x \rightarrow 0} \frac{x^3 - x^2 - 1}{2^x - x^3} (\sin x - \cos x) = 0.$$

98 $y = \frac{1}{2x-3}, \quad y^{(n)}(0) = \frac{1}{3} (-1)^n n! \left(\frac{2}{3}\right)^n.$

$$y = (2x-3)^{-1}, \quad y' = -1 \cdot 2(2x-3)^{-2} y, \quad y'' = 1 \cdot (2) \cdot 2(2x-3)^{-3}$$

$$y^{(n)} = (-1)^n n! 2^n (2x-3)^{-n-1}$$

$$y^{(n)}(0) = \frac{1}{3} (-1)^n n! \left(\frac{2}{3}\right)^n.$$

99 $y = y(x) \quad y \ln y - x - y = 0, \quad y = y(x) \quad (1, 1).$

$$1 \quad y \ln y - x - y = 0 \quad x$$

$$y \ln y - 2y - 1 = 0$$

$$y = \frac{1}{\ln y - 2}$$

$$x \quad y = \frac{\frac{1}{y} y}{(\ln y - 2)^2} = \frac{1}{y(\ln y - 2)^3}.$$

$$x=1, \quad y=1$$

$$y = \frac{1}{8}$$

$$y = x=1, \quad y = \frac{1}{8}, \quad x=1 \quad y < 0, \quad y = y(x) \quad (1,$$

1) .

$$2 \quad y \ln y - x - y = 0 \quad x$$

$$y \ln y - 2y - 1 = 0$$

$$x \quad y \ln y - y - \frac{1}{y} - 2y = 0$$

$$x=1, \quad y=1$$

$$y = \frac{1}{2}, \quad y = \frac{1}{8}$$

$$y = x=1, \quad y = \frac{1}{8}, \quad x=1 \quad y < 0, \quad y = y(x) \quad (1, 1)$$

.

$$3 \quad x \quad y, \quad y \ln y - x - y = 0 \quad y$$

$$x \quad \ln y - y \frac{1}{y} - 1 - \ln y$$

$$y \quad x \quad \frac{1}{y}$$

$$\begin{array}{l} x=1, \quad y=1 \\ x > 0, \end{array} \quad \begin{array}{l} x \quad 1. \\ x = x(y) \quad (1, 1) \end{array}, \quad \begin{array}{l} x \quad y=1 \\ y = y(x) \quad (1, 1) \end{array}, \quad \begin{array}{l} x \quad 1 \\ y=1 \end{array}.$$

$$\begin{array}{l} 100 \\ (a,b) \end{array} \quad \begin{array}{l} f(x), g(x) \quad [a, b] \\ f(\quad) \quad g(\quad). \end{array} \quad (a, b) \quad f(a)=g(a), f(b)=g(b),$$

$$F(x) \quad f(x) \quad g(x) \quad \text{务}$$

$$\begin{array}{l} F(\quad) \quad 0, \\) \end{array} \quad \begin{array}{l} F(x) \\ F(a)=F(b)=0, \end{array} \quad \begin{array}{l} F(x) \\ c \quad (a,b) \end{array} \quad \begin{array}{l} F(x) \\ F(c) \quad 0 \end{array} \quad \begin{array}{l} (\\ [a,c],[c,b] \end{array}$$

$$F(x)$$

$$F(x) \quad f(x) \quad g(x) \quad F(a)=F(b)=0. \quad f(x), g(x) \quad (a, b),$$

$$x_1 \quad x_2, \quad x_1, x_2 \quad (a,b)$$

$$f(x_1) \quad M \quad \max_{[a,b]} f(x), g(x_2) \quad M \quad \max_{[a,b]} g(x)$$

$$x_1 \quad x_2 \quad c \quad x_1, \quad F(c) \quad 0.$$

$$x_1 \quad x_2 \quad F(x_1) \quad f(x_1) \quad g(x_1) \quad 0, F(x_2) \quad f(x_2) \quad g(x_2) \quad 0$$

$$c \quad [x_1, x_2] \quad (a,b) \quad F(c) \quad 0.$$

$$[a,c],[c,b] \quad {}_1 \quad (a,c), \quad {}_2 \quad (c,b)$$

$$F(\quad {}_1) \quad F(\quad {}_2) \quad 0.$$

$$F(x) \quad [\quad {}_1, \quad {}_2] \quad (\quad {}_1, \quad {}_2) \quad (a,b)$$

$$F(\quad) \quad 0 \quad f(\quad) \quad g(\quad)$$