1.

$a\pm b)^2=a^2\pm 2ab+b^2$

$a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc$

$a^2-b^2=(a-b)(a+b)$

$a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$

$a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)$

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

2.

1 $a^m\cdot a^n=a^{m+n}(a>0,a\neq 1)$ 2 $a^m\div a^n=a^{m-n}$

3 $(a^m)^n=a^{mn}$ 4 $(ab)^m=a^mb^m$

5 $(\frac{a}{b})^m=\frac{a^m}{b^m}$ 6 $a^{-m}=\frac{1}{a^m}$

7 $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab},(a\geq 0,b\geq 0)$ 8 $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}},(a\geq 0,b>0)$

3. $\log_a N,a>0,a\neq 1,N>0$

1 $N=a^{\log_a N}$ $N=e^{\ln N}$

2 $\log_a(MN)=\log_a M+\log_a N$

3 $\log_a(\frac{M}{N})=\log_a M-\log_a N$

4 $\log_a(M^n)=n\log_a M$

5 $\log_{a^n}\sqrt[n]{M}=\frac{1}{n}\log_a M$

6 $\log_a M=\frac{\log_b M}{\log_b a}(b>0,b\neq 1)$

7 $\log_a 1 = 0$

$a>b,c<0\Rightarrow ac<bc,\frac{a}{c}<\frac{b}{c}$

$a>b,c<0\Rightarrow ac<bc,\frac{a}{c}<\frac{b}{c}$

12.

1 $a,b\in R\Rightarrow a^2+b^2\geq 2ab$ ($a=b$ “=”)

2 $a,b\in R^+\Rightarrow \frac{a+b}{2}\geq\sqrt{ab}$ ($a=b$ “=”)

3 $a^3+b^3+c^3\geq 3abc(a>0,b>0,c>0)$

4 $(a^2+b^2)(c^2+d^2)\geq(ac+bd)^2,a,b,c,d\in R$

5 $|a|-|b|\leq|a+b|\leq|a|+|b|.$

13.

$a>0$ $|x|<a\Leftrightarrow x^2<a^2\Leftrightarrow -a<x<a$

$|x|>a\Leftrightarrow x^2>a^2\Leftrightarrow x>a$ $x<-a$.

14.

(1) $a>1$, $f(x)$ $g(x)$ ( ) ( )

17. n

$$s_n=\frac{n(a_1+a_n)}{2}=na_1+\frac{n(n-1)}{2}d=\frac{d}{2}n^2+(a_1-\frac{1}{2}d)n$$

18. $$a_n=a_1q^{n-1}=\frac{a_1}{q}\cdot q^n(n\in N^*)$$

n

$$s_n=\begin{cases}\frac{a_1(1-q^n)}{1-q},q\neq 1\\ na_1,q=1\end{cases}\qquad s_n=\begin{cases}\frac{a_1-a_nq}{1-q},q\neq 1\\ na_1,q=1\end{cases}.$$

19. n :

(1)1+2+3+4+…+n= $\frac{n(n+1)}{2}$

(2)1+3+5…+ $(2n-1)=n^2$

(3)2+4+6+…+2n= $n(n+1)$

(4) $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$.

20.

n m_1

m_2 …, n m_n ,

$N=m_1+m_2+\cdots+m_n$.

n m_1

m_2 …, n m_n ,

$N=m_1\times m_2\times\cdots\times m_n$.

21. : $A_n^m=n(n-1)\cdots(n-m+1)=\frac{n}{(n-m)}.(n,m\in N^*,\quad m\leq n)$

: $0!=1$.

22.

$C_n^m=\frac{A_n^m}{A_m^m}=\frac{n(n-1)\cdots(n-m+1)}{1\times2\times\cdots\times m}=\frac{n}{m\cdot(n-m)}$ $n\in N^*, m\in N, m\le n$.

23.

1 $C_n^m=C_n^{n-m}$ 2 $C_n^m=C_{n-1}^m+C_{n-1}^{m-1}$

24. $:(a+b)^n=C_n^0a^n+C_n^1a^{n-1}b+C_n^2a^{n-2}b^2+\cdots+C_n^ra^{n-r}b^r+\cdots+C_n^nb^n$.

25. $T_{r+1}=C_n^ra^{n-r}b^r(r=0\ 1\ 2\quad n)$.

26.

“ a $n\xrightarrow{1}0$ b $0\xrightarrow{1}n$ a b n.”

27.

n ($\frac{n}{2}+1$) $C_n^{\frac{n}{2}}$

n $\frac{n+1}{2}$ $\frac{n+3}{2}$ $C_n^{\frac{n+1}{2}}$.

28.

(1) $C_n^r=C_n^{n-r}$,

(2) $C_n^0+C_n^1+\cdots\cdots C_n^n=2^n$ 2^n

(3) $C_n^0+C_n^2+C_n^4\ldots=C_n^1+C_n^3+C_n^5\ldots=2^{n-1}$.

29. $P(A)=\frac{m}{n}$.

30. A B $p(A+B)=p(A)+p(B)$.

31. A B : $p(A\cdot B)=p(A)\cdot p(B)$.

32.n k

$P_n(k)=C_n^kP^k(1-P)^{n-k}$

33. $a+bi=c+di\Leftrightarrow a=c,b=d$. $a,b,c,d\in R$

34. $z=a+bi$ $|z|=|a+bi|=\sqrt{a^2+b^2}$ $a,b\in R$.

35. $a,b,c,d\in R$

(1) $(a+bi)+(c+di)=(a+c)+(b+d)i$

(2) $(a+bi)-(c+di)=(a-c)+(b-d)i$

(3) $(a+bi)(c+di)=(ac-bd)+(bc+ad)i$

(4) $(a+bi)\div(c+di)=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i$ $(c+di\neq 0)$

36.

$d=|z_1-z_2|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $z_1=x_1+y_1i$ $z_2=x_2+y_2i$.

1.

90

90 180

90

180

360

90

180

180

a b c, $a^2+b^2=c^2$

n $=(n-2)\cdot 180^\circ$

.

2.

ABC a b c

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

$$a^2=b^2+c^2-2bc\cos A$$

$$b^2=c^2+a^2-2ca\cos B$$

$$c^2=a^2+b^2-2ab\cos C.$$

1 $S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c$(h_a h_b h_c a b c)

2 $S=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B.$

$$S=bh=ab\sin\varphi$$

S × $\frac{1}{2}$ ×

$$S=\frac{1}{2}rl=\frac{1}{2}r^2\theta \qquad l=r\theta$$

$S=2\pi RH$ $S=2\pi RH+2\pi R^2$

$V=\pi R^2H.$

$l=\sqrt{R^2+H^2}$ $S=\pi Rl$ $S=\pi Rl+\pi R^2;$

$V=\frac{1}{3}\pi R^2H.$

R— , d—

$S=4\pi R^2$ $V=\frac{4}{3}\pi R^3.$

3. $k=\frac{y_2-y_1}{x_2-x_1}(P_1(x_1,y_1)\quad P_2(x_2,y_2))$

4.

(1) (x_1, y_1) k $y - y_1 = k(x - x_1)$

(2) k y b $y = kx + b$

(3) (x_1, y_1) (x_2, y_2) $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} (y_1 \neq y_2 \quad x_1 \neq x_2)$

(4) x y a b $ab \neq 0$ $\frac{x}{a} + \frac{y}{b} = 1$

(5) : $Ax + By + C = 0$(A B 0).

5. :

(1) $l_1 : y = k_1 x + b_1$ $l_2 : y = k_2 x + b_2$

$l_1 \parallel l_2 \Leftrightarrow k_1 = k_2, b_1 \neq b_2$

$l_1 \perp l_2 \Leftrightarrow k_1 \cdot k_2 = -1$.

(2) $l_1 : A_1 x + B_1 y + C_1 = 0$ $l_2 : A_2 x + B_2 y + C_2 = 0$ A_1 A_2 B_1 B_2

$l_1 \parallel l_2 \Leftrightarrow \frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$

$l_1 \perp l_2 \Leftrightarrow A_1 A_2 + B_1 B_2 = 0$.

6.

(1) $\tan\alpha = \left| \frac{k_2 - k_1}{1 + k_2 k_1} \right|$($l_1 : y = k_1 x + b_1$ $l_2 : y = k_2 x + b_2$ $k_1 k_2 \neq -1$)

(2) $\tan\alpha = \left| \frac{A_1 B_2 - A_2 B_1}{A_1 A_2 + B_1 B_2} \right|$

($l_1 : A_1 x + B_1 y + C_1 = 0$ $l_2 : A_2 x + B_2 y + C_2 = 0$ $A_1 A_2 + B_1 B_2 \neq 0$)

7. l_1 l_2 :

(1) $\tan\alpha = \frac{k_2 - k_1}{1 + k_2 k_1}$ ($l_1 : y = k_1 x + b_1, l_2 : y = k_2 x + b_2, k_1 k_2 \neq -1$)

(2) $\tan\alpha = \dfrac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}$

($l_1 : A_1x + B_1y + C_1 = 0$, $l_2 : A_2x + B_2y + C_2 = 0$, $A_1A_2 + B_1B_2 \neq 0$).

8.

A B (x_1, y_1) (x_2, y_2)

$$d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

9. :

$d = \dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$ ($P(x_0, y_0)$, l $Ax + By + C = 0$).

10.

$l_1 : Ax + By + C_1 = 0$, $l_2 : Ax + By + C_2 = 0$ $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

11. A $(x_1, y_1), B(x_2, y_2)$ $p(x, y)$ p AB p AB

λ $\begin{cases} x = \dfrac{x_1 + \lambda x_2}{1 + \lambda} \\ y = \dfrac{y_1 + \lambda y_2}{1 + \lambda} \end{cases}$ $\lambda = 1$ p AB $\begin{cases} x = \dfrac{x_1 + x_2}{2} \\ y = \dfrac{y_1 + y_2}{2} \end{cases}$.

12. :

ABC $A(x_1, y_1)$ $B(x_2, y_2)$ $C(x_3, y_3)$, ABC

: $G\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.

13. (a, b) r : $P(x_0, y_0)$ $(x - a)^2 + (y - b)^2 = r^2$

$$d = \sqrt{(a - x_0)^2 + (b - y_0)^2}$$

$d > r \Leftrightarrow$ P ; $d = r \Leftrightarrow$ P ; $d < r \Leftrightarrow$ P .

14.

$Ax + By + C = 0$ $(x - a)^2 + (y - b)^2 = r^2$: $d = \dfrac{|Aa + Bb + C|}{\sqrt{A^2 + B^2}}$

$d>r\Leftrightarrow$ $\Leftrightarrow\Delta<0$ $d=r\Leftrightarrow$ $\Leftrightarrow\Delta=0$ $d<r\Leftrightarrow$ $\Leftrightarrow\Delta>0$.

15. O_1 O_2 r_1 r_2 $|O_1O_2|=d$

$d>r_1+r_2$ $\Leftrightarrow$ $\Leftrightarrow 4$ $d=r_1+r_2\Leftrightarrow$ $\Leftrightarrow$

$|r_1-r_2|<d<r_1+r_2\Leftrightarrow$ $\Leftrightarrow 2$ $d=|r_1-r_2|\Leftrightarrow$ $\Leftrightarrow 1$

$0<d<|r_1-r_2|\Leftrightarrow$ $\Leftrightarrow$.

16. .

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b>0).$$

$\{x|-a\le x\le a\}$ $\{x|-b\le y\le b\}$ $=2a$ =2b

2c $a^2=b^2+c^2$ $x=\pm\frac{a^2}{c}$ $e=\frac{c}{a}$.

17. .

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\ (a>0,b>0)\quad \frac{y^2}{a^2}-\frac{x^2}{b^2}=1\ (a>0,b>0).$$

$\{x|x\ge a\quad x\le a\}$ y R $=2a$ =2b 2c

$x=\pm\frac{a^2}{c}$ $e=\frac{c}{a}$

18. .

$y^2=2px,(p>0)$ $(\frac{p}{2},0)$ $|AB|=2p$ $x=-\frac{p}{2}$.

$|CF|=x_0+\frac{p}{2}$ $|CD|=x_1+\frac{p}{2}+x_2+\frac{p}{2}=x_1+x_2+p$.

19. 0

$\sin^2\theta+\cos^2\theta=1$ $\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\tan\theta\cdot cot\theta=1$

20. sin(A+B)=sinAcosB+cosAsinB

sin(A-B)=sinAcosB-sinBcosA

cos(A+B)=cosAcosB-sinAsinB

cos(A-B)=cosAcosB+sinAsinB

tan(A+B)=(tanA+tanB)/(1-tanAtanB)

tan(A-B)=(tanA-tanB)/(1+tanAtanB)

ctg(A+B)=(ctgActgB-1)/(ctgB+ctgA)

ctg(A-B)=(ctgActgB+1)/(ctgB-ctgA)

21. $\tan 2A=2\tan A/(1-\tan^2 A)$

$\operatorname{ctg}2A=(\operatorname{ctg}^2 A-1)/2\operatorname{ctg}a$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$

22. $\sin\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{2}}\quad \cos\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{2}}$

$\tan\frac{A}{2}=\pm\sqrt{\frac{1-\cos A}{1+\cos A}}\quad ctg\frac{A}{2}=\pm\sqrt{\frac{1+\cos A}{1-\cos A}}.$

23. $2\sin A\cos B=\sin(A+B)+\sin(A-B)$

$2\cos A\sin B=\sin(A+B)-\sin(A-B)$

$2\cos A\cos B=\cos(A+B)-\sin(A-B)$

$-2\sin A\sin B=\cos(A+B)-\cos(A-B)$

24. $\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$$\cos A+\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$