02810090 Advanced Microeconomics

Problem Set 3.1

Due Nov, 9th

Q1. Production Technology

Consider a Cobb-Douglas production function $Q = F(K, L) = K^a L^b$ with $a > 0$, $b > 0$, and $a + b = 1$, where Q is the output level and K, L is capital input and labor input level, respectively.

(a) Verify that this production function shows constant returns to scale.

(b) Calculate the output elasticity of labor μ_L and capital μ_K. Do they depend on the specific factor input levels?

(c) Show that the marginal rate of technical substitution (MRTS) between labor and capital only depends on factor input ratio.

(d) How will MRTS change when factor input ratio increases?

Q2. Profit Maximization

Consider a general Cobb-Douglas production function $Q = F(K, L) = K^a L^b$ with $a > 0$, $b > 0$.

(a) Under what conditions (restrictions on a and b) is the production function concave?

Now consider the case when $a = b = 1/4$. Suppose that the firm is a price taker in both factor markets and product market. Price of the product is p, price of capital is r, price of labor is w. Calculate the following:

(b) The optimal capital-labor input ratio.

(c) The optimal capital input K^*.

(d) How would optimal captial input change as r increases, w increases, or p increases? (Sign $\frac{dK^*}{dr}$, $\frac{dK^*}{dw}$, and $\frac{dK^*}{dp}$)

Problem Set 2.1, Q1 revised edition

Suppose that a person has utility function of the form:

$$U = y + 2l^{1/2}$$

where y is consumption, and l is leisure. The maximum leisure that can be consumed is 24 hours. The wage rate is w, and the price of consumption is p.

a) Derive the individual's labor supply function.

b) What is the minimum wage at which she is willing to work?

c) How will her supply of hours respond to a small cut in a proportional income tax?

d) How will her supply of $h =$ [illegible] respond to a small cut in a proportional consumption tax?

(For part c) and d), **just answering "increase" or "decrease" is enough**.)