2013

1 8 4 32

$\cos x-1=x\sin\alpha(x),|\alpha(x)|<\frac{\pi}{2}$ $x\to 0$ $\alpha\left(x\right)$

A x **B** x

C x **D** x

$x\to 0$ $\cos x-1=x\sin\alpha(x)\sim-\frac{1}{2}x^2,\sin\alpha(x)\sim-\frac{1}{2}x\sim\alpha(x)$ **C**

2 $y=f\left(x\right)$ $\cos\left(xy\right)-\ln y+x=1$ $\lim\limits_{n\to\infty}n\left(f\left(\frac{2}{n}\right)-1\right)=$

A **2** **B** **1** **C** **-1** **D** **-2**

$x=0$ $y=f(0)=1$ $-\sin(xy)(y+xy')-\frac{y'}{y}+1=0$

$x=0,y=1$ $y'(0)=f'(0)=1$

$$\lim_{n\to\infty}n\left(f\left(\frac{2}{n}\right)-1\right)=2\lim_{n\to\infty}\frac{f(\frac{2}{n})-f(0)}{\frac{2}{n}}=2f'(0)=2$$ **A**

$f(x)=\begin{cases}\sin x,x\in[0,\pi)\\ 2,\quad x\in[\pi,2\pi]\end{cases}$ $F(x)=\int_0^x f(t)dt$

$x=\pi$ $F(x)$ $x=\pi$ $F(x)$

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$x=\pi$ $f(x)$ $F(x)=\int_0^x f(t)dt$

$f(x)=\begin{cases}\dfrac{1}{(x-1)^{\alpha-1}},1<x<e\\ \dfrac{1}{x\ln^{\alpha+1}x},x\geq e\end{cases}$ $\int^{+\infty}f\left(x\right)dx$

A $\alpha<-2$ **B** $a>2$ **C** $-2<a<0$ **D** $0<\alpha<2$

$$\int_1^{+\infty}f(x)dx=\int_1^e\frac{dx}{(x-1)^{\alpha-1}}+\int_e^{+\infty}\frac{1}{x\ln^{\alpha+1}x}dx$$

$\int_1^e\frac{dx}{(x-1)^{\alpha-1}}=\int_0^{e-1}\frac{dt}{t^{\alpha-1}}$ $\alpha-1<1$

$$\int_e^{+\infty} \frac{1}{x\ln^{\alpha+1} x} dx = -\frac{1}{\alpha} \ln^{-\xi} x \,|_1^{+\infty} = -\frac{1}{\alpha} \frac{1}{\lim\limits_{x\to+\infty} \ln^{\alpha} x} \qquad a > 0$$

$0 < \alpha < 2$ $\int^{+\infty} f(x)dx$

$z = \frac{y}{x} f(xy)$ f $-\frac{\partial}{\partial} + \frac{\partial}{\partial} =$

A $2yf'(xy)$ **B** $-2yf'(xy)$ **C** $\frac{2}{x} f(xy)$ **D** $-\frac{2}{x} f(xy)$

$$\frac{x}{y}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{x}{y}\left(-\frac{y}{x^2} f(xy) + \frac{y^2}{x} f'(xy)\right) + \frac{1}{x} f(xy) + yf'(xy) = 2yf'(xy)$$ **A**

6 D_k k $I_k = \iint\limits_{D_k} (y-x)dxdy$

C $a=2,b=0$ D $a=2$ b

$$\begin{pmatrix}2&0&0\\0&b&0\\0&0&0\end{pmatrix}\qquad \mathbf{A}=\begin{pmatrix}1&a&1\\a&b&a\\1&a&1\end{pmatrix}\qquad \begin{pmatrix}2&0&0\\0&b&0\\0&0&0\end{pmatrix}$$

$$|\lambda E-A|=\begin{vmatrix}\lambda-1&-a&-1\\-a&\lambda-b&-a\\-1&-a&\lambda-1\end{vmatrix}=-\lambda(\lambda^2-(b+2)\lambda+2b-2a^2)$$

$2b-2a^2=2b$ $a=0$ b **B**

6 4 24 .

9 $\lim\limits_{x\to0}\left(2-\frac{\ln(1+x)}{x}\right)^{\frac{1}{x}}=$________

$$\lim_{x\to0}\left(2-\frac{\ln(1+x)}{x}\right)^{\frac{1}{x}}=\lim_{x\to0}\left(1+\frac{x-\ln(1+x)}{x}\right)^{\frac{1}{x}}=e^{\lim\limits_{x\to0}\frac{x-\ln(1+x)}{x^2}}=e^{\lim\limits_{x\to0}\frac{x-(x-\frac{1}{2}x^2+o(x^2))}{x^2}}=e^{\frac{1}{2}}$$

10 $f(x)=\int_{-1}^{x}\sqrt{1-e^t}\,dt$ $y=f(x)$ $x=f^{-1}(y)$ $y=0$ $\frac{dx}{dy}|_{y=0}=$______

$$\frac{dx}{dy}|_{y=0}=\frac{1}{\frac{dy}{dx}|_{x=-1}}=\frac{1}{\sqrt{1-e^{-1}}}$$

11 L $=\cos3\theta\left(-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}\right)$ t L

____

$$A=\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}r^2d\theta=\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2 3\theta d\theta=\frac{1}{3}\int_{0}^{\frac{\pi}{2}}\cos^2 tdt=\frac{\pi}{12}$$

$\frac{\pi}{12}$

12 $\begin{cases}x=\arctan t\\y=\ln\sqrt{1+t^2}\end{cases}$ $t=1$ ______________

$t=1$ $x=\frac{\pi}{4},y=\frac{1}{2}\ln2$ $y'|_{t=1}=\frac{\frac{t}{1+t^2}}{\frac{1}{1+t^2}}|_{t=1}=1$

$y-\frac{1}{2}\ln2=-1(x-\frac{\pi}{4})$ $y+x-\frac{1}{2}\ln2-\frac{\pi}{4}=0$

13 $y_1 = e^{3x} - xe^{2x}, y_2 = e^x - xe^{2x}, y_3 = -xe^{2x}$

$y(0) = 0, y'(0) = 1$ ________________

$y_1 - y_3 = e^{3x}$ $y_2 - y_3 = e^x$

$y = C_1 e^{3x} + C_2 e^x - xe^{2x}$ C_1, C_2

$C_1 = 1, C_2 = -1$ $y = e^{3x} - e^x - xe^{2x}$

14 $A = (a_{ij})$ $|A|$ A_{ij} a_{ij}

$A_{ij} + a_{ij} = 0 (i, j = 1,2,3)$ $|A| =$ ______________

$A_{ij} + a_{ij} = 0 (i, j = 1,2,3)$ $A + A^{*T} = 0$ A^* **A**

$|A^*| = |A^{*T}| = |A|^{3-1} = -|A|$ $|A|$ -1 **0**

$$r(A^*) = \begin{cases} n, r(A) = n \\ 1, r(A) = n-1 \\ 0, r(A) < n-1 \end{cases}$$ $A + A^{*T} = 0$ $r(A) = r(A^*)$, **3**

$|A| = -1.$

15 **10**

$x \to 0$ $1 - \cos x \cos 2x \cos 3x$ ax^n a, n

$x \to 0$

$x \to 0$ $\cos x = 1 - \frac{1}{2}x^2 + o(x^2)$ $\cos 2x = 1 - \frac{1}{2}(2x)^2 + o(x^2) = 1 - 2x^2 + o(x^2)$

$\cos 3x = 1 - \frac{1}{2}(3x)^2 + o(x^2) = 1 - \frac{9}{2}x^2 + o(x^2)$

$$1 - \cos x \cos 2x \cos 3x = 1 - (1 - \frac{1}{2}x^2 + o(x^2))(1 - 2x^2 + o(x^2))(1 - \frac{9}{2}x^2 + o(x^2)) = 7x^2 + o(x^2)$$

$1 - \cos x \cos 2x \cos 3x$ ax^n $a = 7, n = 2$

16 **10**

D $y = \sqrt[3]{x}$ $x = a \ (a > 0)$ x V_x, V_y **D** x y

$10V_x = V_y$ a

$$V_x = \pi \int_0^a y^2 dx = \pi \int_0^a x^{\frac{2}{3}} dx = \frac{3}{5} a^{\frac{5}{3}} \pi$$

$$V_y = 2\pi\int_0^a xf(x)dx = 2\pi\int_0^a x^{\frac{4}{3}}dx = \frac{6}{7}a^{\frac{7}{3}}\pi$$

$10V_x = V_y$ $a = 7\sqrt{7}$

17 **10**

D $x = 3y, y = 3x, x + y = 8$ $\iint\limits_D x^2 dxdy$

$$\iint\limits_D x^2 dxdy = \iint\limits_{D_1} x^2 dxdy + \iint\limits_{D_2} x^2 dxdy = \int_0^2 x^2 dx\int_{\frac{x}{3}}^{3x} dy + \int_2^6 x^2 dx\int_{\frac{x}{3}}^{8-x} dy = \frac{416}{3}$$

18 **10**

$f(x)$ $[-1,1]$ $f(1) = 1$

1 $\xi \in (0,1)$ $f'(\xi) = 1$

2 $\eta \in (-1,1)$ $f''(\eta) + f'(\eta) = 1$

1 $f(x)$ $f(0) = 0$ $f(x)$ $[-1,1]$

$\xi \in (0,1)$ $f'(\xi) = \dfrac{f(1) - f(0)}{1 - 0} = 1$

2 $f(x)$ $f'(x)$ **1** $\xi \in (0,1)$ $f'(\xi) = 1$ $f'(-\xi) = 1$

$\varphi(x) = e^x(f'(x) - 1)$ $\varphi(x)$ $[-1,1]$ $\varphi(-\xi) = \varphi(\xi) = 0$

$\eta \in (-\xi, \xi) \subset (-1,1)$ $\varphi'(\eta) = 0,$ $f''(\eta) + f'(\eta) = 1$

19 **10**

$x^3 - xy + y^3 = 1(x \geq 0, y \geq 0)$

$$L(x,y) = x^2 + y^2 + \lambda(x^3 - xy + y^3 - 1)$$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 2x + \lambda(3x^2 - y) = 0 \\ \dfrac{\partial L}{\partial y} = 2y + \lambda(3y^2 - x) = 0 \\ x^3 - xy + y^3 = 1 \end{cases}$$ $x = 1, y = 1$ $M_1(1,1)$

$M_2(0,1), M_3(1,0)$

$f(x,y)=\sqrt{x^2+y^2}$ $f(1,1)=\sqrt{2}, f(0,1)=1, f(1,0)=1$

$\sqrt{2}$ **1**

20 **11**

$f(x)=\ln x+\frac{1}{x}$

$f(x)$

$\{x_n\}$ $\ln x_n+\frac{1}{x_{n+1}}<1$ $\lim\limits_{n\to\infty} x_n$

1 $f'(x)=\frac{1}{x}-\frac{1}{x^2}=\frac{x-1}{x^2}$

$f'(x)=0$ $x=1$

$x\in(0,1)$ $f'(x)<0$ $x\in(1,\infty)$ $f'(x)>0$

$x=1$ $f(1)=1$

2 $\ln x_n+\frac{1}{x_{n+1}}<1$ $\ln x_n+\frac{1}{x_n}\geq 1$ $\frac{1}{x_{n+1}}<\frac{1}{x_n}$ $\{x_n\}$

$\ln x_n\leq \ln x_n+\frac{1}{x_{n+1}}<1$ $0<x_n<e$ $\{x_n\}$

$\lim\limits_{n\to\infty} x_n$

$\lim\limits_{n\to\infty} x_n=a$ $\lim\limits_{n\to\infty}\left(\ln x_n+\frac{1}{x_{n+1}}\right)=\ln a+\frac{1}{a}\leq 1$ **1** $\lim\limits_{n\to\infty} x_n=a=1$

21 **11**

L $y=\frac{1}{4}x^2-\frac{1}{2}\ln x(1\leq x\leq e)$

1 **L**

2

2 $\left(\bar{x},\bar{y}\right)$

$$\bar{x}=\frac{\iint\limits_D xdxdy}{\iint\limits_D dxdy}=\frac{\int_1^e xdx\int_0^{\frac{1}{4}x^2-\frac{1}{2}\ln x}dy}{\int_1 dx\int_0^{\frac{1}{4}x^2-\frac{1}{2}\ln x}dy}=\frac{\frac{e^4-2e^2-3}{16}}{\frac{e^3-7}{12}}=\frac{3(e^4-2e^2-3)}{4(e^3-7)}$$

22 **11**

$A=\begin{pmatrix}1 & a\\ 1 & 0\end{pmatrix},B=\begin{pmatrix}0 & 1\\ 1 & b\end{pmatrix}$ a,b **C** $AC-CA=B$ **C**

$AC-CA=B$ **C** **2** $C=\begin{pmatrix}x_1 & x_2\\ x_3 & x_4\end{pmatrix}$

$AC-CA=B$ $\begin{pmatrix}-x_2+ax_3 & -ax_1+x_2+ax_4\\ x_1-x_3-x_4 & x_2-ax_3\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & b\end{pmatrix}$

$$\begin{cases}-x_2+ax_3=0\\ -ax_1+x_2+ax_4=1\\ x_1-x_3-x_4=1\\ x_2-ax_3=b\end{cases}$$ **C**

$$\left(A\,|\,b\right)=\begin{pmatrix}0 & -1 & a & 0 & 0\\ -a & 1 & 0 & a & 1\\ 1 & 0 & -1 & -1 & 1\\ 0 & 1 & -a & 0 & b\end{pmatrix}\to\begin{pmatrix}1 & 0 & -1 & -1 & 1\\ 0 & 1 & -a & 0 & 0\\ 0 & 0 & 0 & 0 & 1+a\\ 0 & 0 & 0 & 0 & b\end{pmatrix}$$

$a=-1,b=0$ **C** $AC-CA=B$

$$\left(A\,|\,b\right)\to\begin{pmatrix}1 & 0 & -1 & -1 & 1\\ 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$x=\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix}+C_1\begin{pmatrix}1\\ -1\\ 1\\ 0\end{pmatrix}+C_2\begin{pmatrix}1\\ 0\\ 0\\ 1\end{pmatrix}$$ $AC-CA=B$ **C**

$C=\begin{pmatrix}1+C_1+C_2 & -C_1\\ C_1 & C_2\end{pmatrix}$ C_1,C_2

23 **11**

$$f(x_1,x_2,x_3)=2(a_1x_1+a_2x_2+a_3x_3)^2+(b_1x_1+b_2x_2+b_3x_3)^2 \qquad \alpha=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix},\beta=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

1 f $2\alpha\alpha^T+\beta\beta^T$

2 α,β f $2y_1^2+y_2^2$

1

$$\begin{aligned}f(x_1,x_2,x_3)&=2(a_1x_1+a_2x_2+a_3x_3)^2+(b_1x_1+b_2x_2+b_3x_3)^2\\
&=2(x_1,x_2,x_3)\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}(a_1,a_2,a_3)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+(x_1,x_2,x_3)\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}(b_1,b_2,b_3)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\\
&=(x_1,x_2,x_3)(2\alpha\alpha^T)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+(x_1,x_2,x_3)(\beta\beta^T)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\\
&=(x_1,x_2,x_3)(2\alpha\alpha^T+\beta\beta^T)\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\end{aligned}$$

f $2\alpha\alpha^T+\beta\beta^T$

2 $A=2\alpha\alpha^T+\beta\beta^T$ $|\alpha|=1,\beta^T\alpha=0$

$A\alpha=(2\alpha\alpha^T+\beta\beta^T)\alpha=2\alpha|\alpha|^2+\beta\beta^T\alpha=2\alpha$ α $\lambda_1=2$

$A\beta=(2\alpha\alpha^T+\beta\beta^T)\beta=2\alpha\alpha^T\beta+\beta|\beta|^2=\beta$ β $\lambda_2=1$

A $r(A)=r(2\alpha\alpha^T+\beta\beta^T)\le r(2\alpha\alpha^T)+r(\beta\beta^T)=2$ $\lambda_3=0$

f $2y_1^2+y_2^2$