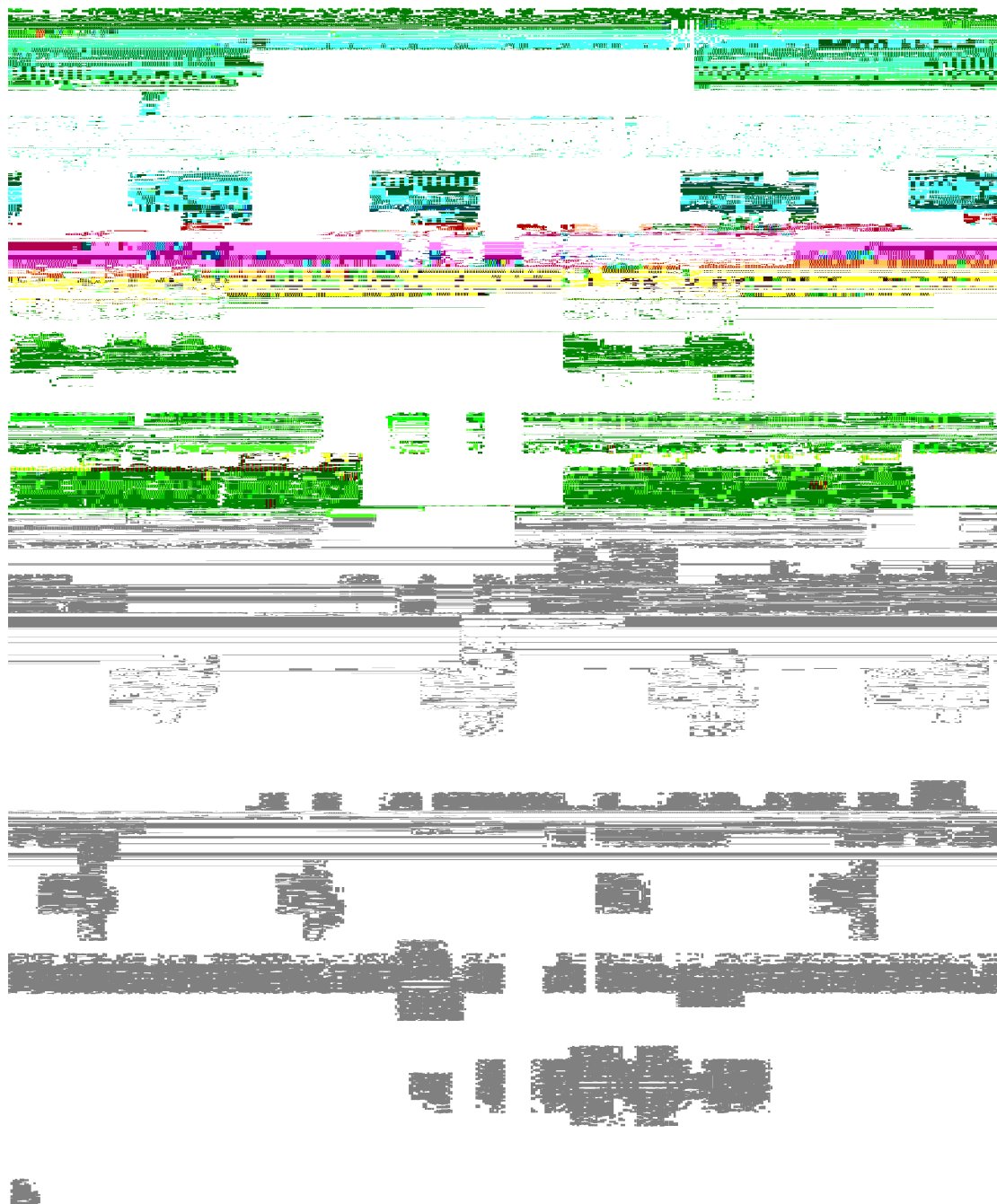


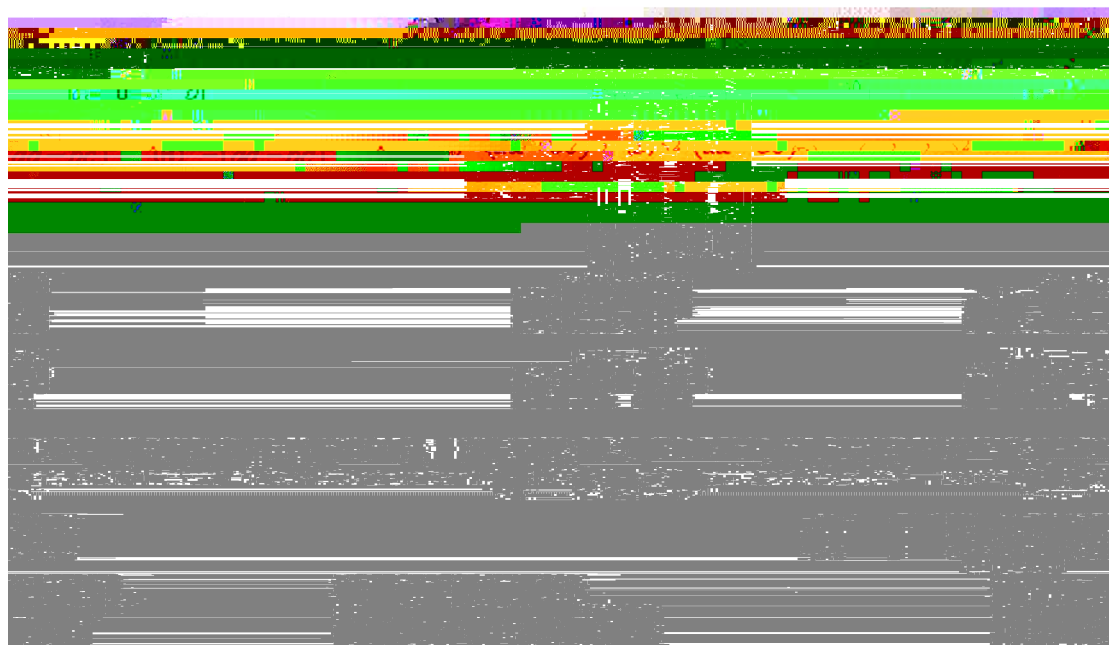
2014 年全国硕士研究生入学统一考试数学一试题



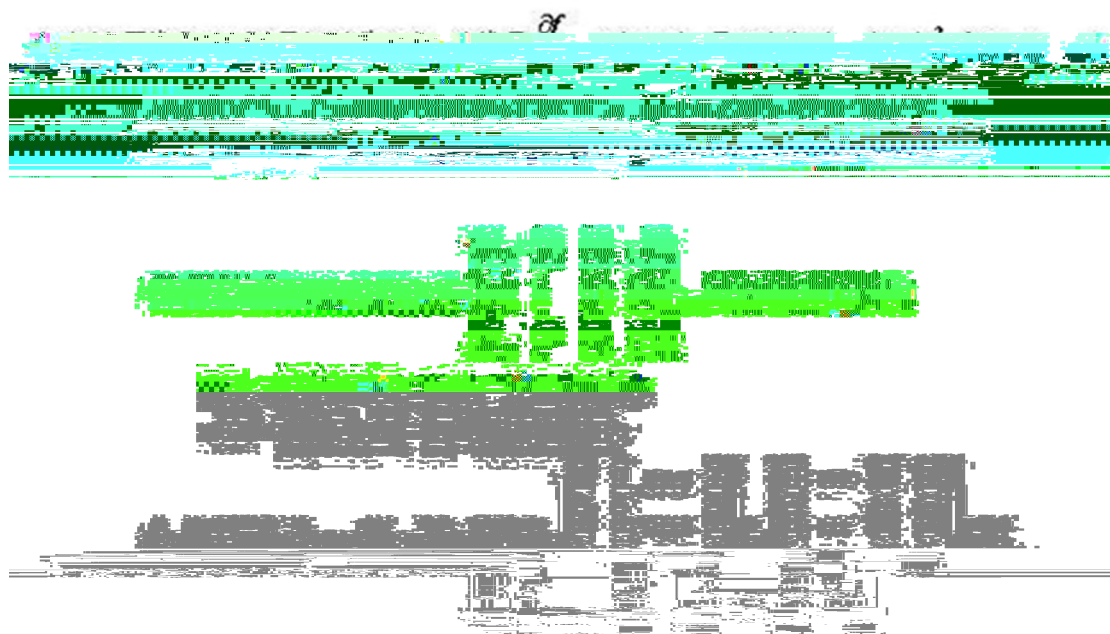
(A) $u(x, y)$ 的最大值和最小值都在 D 的边界上取得

..... 在 D 的内部上取得

(C) 取得, 最小值在 D 的边界上取得



及 x 轴所围成平面图形的面积, 求极限 $\lim_{n \rightarrow \infty} nS_n$.



2014 年全国硕士研究生入学统一考试 数学二试题答案

一、选择题：1~8 小题，每小题 4 分，共 32 分，下列每小题给出的四个选项中，只有一项符合题目要求的，请将所选项前的字母填在答题纸指定位置上。

(1) B

(2) B

(3) D

(4) C

(5) D

(6) A

(7) B

(8) A

二、填空题：9~14 小题，每小题 4 分，共 24 分，请将答案写在答题纸指定位置上。

(9) $\frac{3}{8}$

(10) $1 \quad 1$

(11)

(12) $\frac{2}{\quad} \quad \frac{\quad}{2}$

(13) $\frac{11}{20}$

(14) $[-2, 2]$

三、解答题：15~23 小题，共 94 分。请将解答写在答题纸指定位置上。解答应写出文字说明、证明过程或演算步骤。

(15) 【答案】

1时，取极小值 $\frac{1}{2}$ 0。

(17) 【答案】

$$\frac{1}{2} \frac{1}{0} \frac{2}{1} \frac{1}{1}$$

$$\frac{1}{2} \frac{1}{0} \frac{2}{1} \frac{1}{1}$$

$$\frac{1}{2} \frac{1}{0} \frac{2}{1} \frac{1}{1}$$

$$\frac{1}{2} \frac{1}{0} \frac{2}{1} \frac{1}{1}$$

$$\frac{1}{2} \frac{1}{0} \frac{2}{1} \frac{1}{1}$$

$$\frac{3}{2} \frac{1}{0} \frac{2}{1}$$

$$\frac{3}{4}$$

(18) 【答案】

—

(19) 【答案】

证明：1) 因为 $0 < x < 1$ ，所以有定积分比较定理可知， $0 < \int_0^1 x dx < \int_0^1 1 dx = 1$ ，即

$$0 < \int_0^1 x dx < 1.$$

2) 令

$$f(x) = \int_0^1 x dx$$

由 1) 可知 $f(x) > 0$ ，

所以 $f(x)$ 是单调递增的。

由 $f(x)$ 是单调递增，可知

$$f(x) > 0$$

由因为 $0 < x < 1$ ，所以 $0 < \int_0^1 x dx < 1$ ， $f(x)$ 单调递增，所以 $f(x) > 0$ ，得证。

(20) 【答案】

因为

$$1 = \frac{1}{1} = 0.1$$

$$2 = \frac{1}{1} = \frac{1}{1} = 0.1$$

∴

$$1 = \frac{1}{1}$$

所以

$$\frac{1}{1} = \frac{1}{1} = 1$$

$$\frac{1}{1} = \frac{1}{1} = 1$$

$$\frac{1}{1} = \frac{1}{1} = 1$$

$$\frac{1}{1} = \frac{1}{1} = 1$$

所以

$$\frac{1}{-1} \quad \frac{1}{-1}$$

$$1 \quad \frac{1}{-1} \quad 0$$

(21) 【答案】

(22) 【答案】 ① $1, 2, 3, 1$ ②

$$\begin{array}{ccccccc} & & 1 & 2 & & 2 & 6 & & 3 & 1 \\ 2 & 1 & 1 & 2 & 2 & 3 & 2 & 3 & 1 \\ 3 & 1 & 1 & 3 & 2 & 4 & 3 & 3 & 1 & 1', 2', 3' \end{array}$$

(23) 【答案】利用相似对角化的充要条件证明。

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1.B

$$\lim_{x \rightarrow 0^+} \frac{\ln^\alpha(1+2x)}{x} = \lim_{x \rightarrow 0^+} \frac{(2x)^\alpha}{x} = 2^\alpha \lim_{x \rightarrow 0^+} x^{\alpha-1} = 0$$

$$\lim_{x \rightarrow 0^+} \frac{(1-\cos x)^{\frac{1}{2}}}{x} = \lim_{x \rightarrow 0^+} \frac{(\frac{1}{2}x^2)^{\frac{1}{2}}}{x} = (\frac{1}{2})^{\frac{1}{2}} \lim_{x \rightarrow 0^+} x^{\frac{2}{2}-1} = 0$$

$$\therefore \frac{2}{\alpha} - 1 > 0 \therefore \alpha < 2$$

□ □

$$y = x + \sin \frac{1}{x}$$

$$k = \lim_{x \rightarrow \infty} \frac{y}{x} = \lim_{x \rightarrow \infty} \frac{x + \sin \frac{1}{x}}{x} = 1$$

$$\lim_{x \rightarrow \infty} (y - x) = \lim_{x \rightarrow \infty} \sin \frac{1}{x} = 0$$

$$\therefore y = x + \sin \frac{1}{x} \quad y = x$$

□ □

$$f(x) = x^2, \quad [0,1]$$

$$f(0) = 0$$

$$f(1) = 1$$

$$\therefore g(x) = 0 \cdot (1-x) + 1 \cdot x = x$$

$$\therefore f(x) \leq g(x)$$

$$f''(x) = 2 \geq 0 \therefore D$$

4 C

$$\frac{dy}{dx} = \frac{2t+4}{2t}$$

$$\left. \frac{dy}{dx} \right|_{t=1} = 3$$

$$\frac{d^2 y}{dx^2} = \frac{2 \cdot 2t - 2(2t+4)}{(2t)^2} = \frac{-8}{(2t)^3}$$

$$\therefore \left. \frac{d^2 y}{dx^2} \right|_{t=1} = -1$$

$$k = \frac{|y''|}{(1+y'^2)^{\frac{3}{2}}} = \frac{1}{(1+3^2)^{\frac{3}{2}}}$$

$$\therefore R = \frac{1}{k} = (1+3^2)^{\frac{3}{2}} = 10^{\frac{3}{2}} = 10\sqrt{10}$$

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$$\frac{f(x)}{x} = \frac{\arctan x}{x} = \frac{1}{1+\xi^2} \cdot \xi^2 = \frac{x - \arctan x}{\arctan x}.$$

$$\lim_{x \rightarrow 0} \frac{\xi^2}{x^2} = \lim_{x \rightarrow 0} \frac{x - \arctan x}{x^2 \cdot \arctan x} = \lim_{x \rightarrow 0} \frac{x - \arctan x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x^2}}{3x^2} = \lim_{x \rightarrow 0} \frac{x^2}{3x^2(1+x^2)} = \frac{1}{3} \quad \square$$

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$$B = \frac{\partial^2 u}{\partial x \partial y} > 0 \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad A = \frac{\partial^2 u}{\partial x^2} \quad B = \frac{\partial^2 u}{\partial y^2} \quad \square$$

$$AC - B^2 < 0 \quad u(x, y) \quad D \quad \square$$

$$D \quad \square$$

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$$\begin{vmatrix} 0 & a & b & 0 \\ a & 0 & 0 & b \\ 0 & c & d & 0 \\ c & 0 & 0 & d \end{vmatrix} \\
 = a \times (-1)^{2+1} \begin{vmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & d \end{vmatrix} + c \times (-1)^{4+1} \begin{vmatrix} a & b & 0 \\ 0 & 0 & b \\ c & d & 0 \end{vmatrix} \\
 = -a \times d \times (-1)^{3+3} \begin{vmatrix} a & b \\ c & d \end{vmatrix} - c \times b \times (-1)^{2+3} \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\
 = -ad \begin{vmatrix} a & b \\ c & d \end{vmatrix} + bc \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\
 = (bc - ad) \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\
 = -(ad - bc)^2$$

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$$\begin{matrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \lambda_1 & \alpha_1 + k\alpha_3 & \lambda_2(\alpha_2 + l\alpha_3) = 0 \\ \lambda_1\alpha_1 + \lambda_2\alpha_2 + k\lambda_1 + l\lambda_2\alpha_3 = 0 \\ \Rightarrow \lambda_1 = \lambda_2 = k\lambda_1 + l\lambda_2 = 0 \end{matrix}$$

$$\alpha_1 + k\alpha_3 \quad \alpha_2 + l\alpha_3$$

$$\alpha_1 + k\alpha_3 \quad \alpha_2 + l\alpha_3$$

$$\alpha_1 \quad \alpha_2 \quad \alpha_3$$

$$\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \alpha_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$9. \int_{-\infty}^1 \frac{1}{x^2 + 2x + 5} dx = \int_{-\infty}^1 \frac{1}{(x+1)^2 + 4} dx = \frac{1}{2} \arctan \frac{x+1}{2} \Big|_{-\infty}^1 = \frac{1}{2} \left[\frac{\pi}{4} - \left(-\frac{\pi}{2} \right) \right] = \frac{3}{8} \pi$$

10.

$$f'(x) = 2(x-1)x \in [0, 2]$$

$$\therefore f(x) = x^2 - 2x + c$$

$$f(x)$$

$$\therefore f(0) = 0 \therefore c = 0$$

$$\therefore f(x) = x^2 - 2x$$

$$x \in [0, 2]$$

$$f(x) \quad \mathbf{4}$$

$$\therefore f(7) = f(3) = f(-1) = -f(1) = -(1-2) = 1$$

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□

$$e^{2yz}(2y \cdot \frac{\partial z}{\partial x}) + 2x + \frac{\partial z}{\partial x} = 0$$

$$x = \frac{1}{2}, y = \frac{1}{2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{e^{z \cdot \frac{1}{2} \cdot \frac{1}{2}} + 1}$$

y

$$e^{2yz}(2z + 2y \frac{\partial z}{\partial y}) + 2y + \frac{\partial z}{\partial y} = 0$$

$$x = \frac{1}{2}, y = \frac{1}{2}$$

$$\frac{\partial z}{\partial y} = \frac{1 - z \cdot \frac{1}{2} \cdot \frac{1}{2} e^{z \cdot \frac{1}{2} \cdot \frac{1}{2}}}{e^{z \cdot \frac{1}{2} \cdot \frac{1}{2}} + 1}$$

12.

$$\begin{cases} x = r \cos \theta = \theta \cos \theta \\ y = r \sin \theta = \theta \sin \theta \end{cases}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin \theta + \theta \cos \theta}{\cos \theta - \theta \sin \theta}$$

$$\left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{2}} = \frac{1 + \frac{\pi}{2} \cdot 0}{0 - \frac{\pi}{2} \cdot 1} = -\frac{2}{\pi}$$

$$\theta = \frac{\pi}{2} \quad \begin{cases} x = \theta \cos \theta = 0 \\ y = \theta \sin \theta = \frac{\pi}{2} \end{cases}$$

$$(y - \frac{\pi}{2}) = -\frac{2}{\pi}(x - 0)$$

$$y = -\frac{2}{\pi}x + \frac{\pi}{2}$$

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$$\frac{\int_0^1 x f(x) dx}{\int_0^1 f(x) dx} = \frac{\int_0^1 x(-x^2 + 2x + 1) dx}{\int_0^1 (-x^2 + 2x + 1) dx} = \frac{\left(-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^1}{\left(-\frac{1}{3}x^3 + x^2 + x\right)\Big|_0^1} = \frac{11}{20} \quad \square$$

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$$\begin{aligned} f(x_1, x_2, x_3) &= x_1^2 - x_2^2 + 2ax_1x_3 + 4x_2x_3 \\ &= (x_1 + ax_3)^2 - (x_2 - 2x_3)^2 + 4x_3^2 - a^2x_3^2 \end{aligned}$$

$$\therefore f \quad 1$$

$$\therefore 4 - a^2 \geq 0$$

$$\therefore -2 \leq a \leq 2$$

15.

$$\lim_{x \rightarrow \infty} \frac{\int_1^x (t^2(e^{\frac{1}{t}} - 1) - t) dt}{x^2 \ln(1 + \frac{1}{x})} = \lim_{x \rightarrow \infty} \frac{\int_1^x (t^2(e^{\frac{1}{t}} - 1) - t) dt}{x^2 \cdot \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{x^2(e^{\frac{1}{x}} - 1) - x}{1} = \lim_{x \rightarrow \infty} x^2(e^{\frac{1}{x}} - 1 - \frac{1}{x})$$

$$\underline{\underline{\frac{1}{x} = t}} = t \lim_{x \rightarrow \infty} \frac{e^t - 1 - t}{t^2} = \lim_{x \rightarrow \infty} \frac{1 + t + \frac{1}{2}t^2 + O(t^2) - 1 - t}{t^2} = \frac{1}{2}$$

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$$\because x^2 + y^2 y' = 1 - y'$$

$$\therefore y' = \frac{1 - x^2}{y^2 + 1} \quad \square$$

$$y' = 0, \therefore x = \pm 1$$

$$\therefore y'' = \frac{-2x(y^2 + 1) - (1 - x^2) \cdot 2yy'}{(y^2 + 1)^2} \quad \square$$

$$\because y'(1) = y'(-1) = 0$$

$$\therefore y''(1) = \frac{-2}{y^2(1) + 1} < 0, \therefore y(1) \quad \square$$

$$y''(-1) = \frac{2}{y^2(1) + 1} > 0, y(-1)$$

$$\because y' = \frac{1 - x^2}{y^2 + 1}, \therefore (y^2 + 1)dy = (1 - x^2)dx, \therefore \int (y^2 + 1)dy = \int (1 - x^2)dx$$

$$\therefore \frac{1}{3}y^3 + y = x - \frac{1}{3}x^3 + c \quad \square$$

$$y(2) = 0$$

$$\therefore c = \frac{2}{3}$$

$$\therefore \frac{1}{3}y^3 + y = x - \frac{1}{3}x^3 + \frac{2}{3} \quad \square$$

$$x=1$$

$$\therefore \frac{1}{3}y^3(1) + y(1) = 1 - \frac{1}{3} + \frac{2}{3}$$

$$\therefore y(1) = 1$$

$$x=-1,$$

$$\therefore \frac{1}{3}y^3(-1) + y(-1) = -1 + \frac{1}{3} + \frac{2}{3} = 0$$

$$\therefore y(-1) = 0$$

□

17

$$D \quad y=x$$

$$\begin{aligned} \iint_D \frac{x \sin(\pi \sqrt{x^2 + y^2})}{x+y} dx dy &= \iint_D \frac{y \sin(\pi \sqrt{x^2 + y^2})}{x+y} dx dy \\ &= \frac{1}{2} \iint_D \frac{x \sin(\pi \sqrt{x^2 + y^2})}{x+y} + \frac{y \sin(\pi \sqrt{x^2 + y^2})}{x+y} dx dy \\ &= \frac{1}{2} \iint_D \sin(\pi \sqrt{x^2 + y^2}) dx dy \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \int_0^{\frac{\pi}{2}} d\theta \int_1^2 \sin(\pi r) r dr = -\frac{1}{4} \int_1^2 r d \cos(\pi r) \\ &= -\frac{1}{4} r \cos(\pi r) \Big|_1^2 + \frac{1}{4} \int_1^2 \cos(\pi r) dr \\ &= -\frac{1}{2} - \frac{1}{4} = -\frac{3}{4} \end{aligned}$$

□ □ □

$$\frac{\partial z}{\partial x} = f' \cdot e^x \cdot \cos y,$$

$$\frac{\partial^2 z}{\partial x^2} = \cos y \cdot (f'' \cdot e^x \cdot \cos y \cdot e^x + f' \cdot e^x) = f'' \cdot (e^x \cdot \cos y)^2 + f' \cdot e^x \cdot \cos y$$

$$\frac{\partial z}{\partial y} = f' \cdot e^x \cdot (-\sin y),$$

$$\frac{\partial^2 z}{\partial y^2} = -e^x [f'' \cdot e^x \cdot (-\sin y) + f' \cdot \cos y] = (e^x)^2 \sin y^2 f'' - f' \cdot \cos y \cdot e^x$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f'' \cdot e^{2x} = (4z + e^x \cdot \cos y) e^{2x}$$

$$\therefore f''(e^x \cdot \cos y) = 4f(e^x \cdot \cos y) + e^x \cdot \cos y$$

$$t = e^x \cdot \cos y, \therefore f''(t) = 4f(t) + t$$

$$\therefore y'' - 4y = x$$

□

$$\lambda^2 - 4 = 0 \quad \therefore x = \pm 2 \quad \therefore \widetilde{y(x)} = C_1 e^{2x} + C_2 e^{-2x}$$

$$y^* = (ax + b) \quad \therefore y^* = -\frac{1}{4}x$$

$$\therefore y = \widetilde{y(x)} + y^* = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{4}x$$

$$y(0) = 0 = C_1 + C_2$$

$$y'(0) = 0 = 2C_1 - 2C_2 - \frac{1}{4}$$

$$\therefore \begin{cases} C_1 + C_2 = 0 \\ 2C_1 - 2C_2 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{16} \\ C_2 = -\frac{1}{16} \end{cases}$$

$$\therefore f(\mu) = \frac{1}{16} e^{2\mu} - \frac{1}{16} e^{-2\mu} - \frac{1}{4}\mu$$

□

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19.

I

$$h_1(x) = \int_a^x g(t) dt$$

$$h_1(a) = 0$$

$$h_1'(x) = g(x) \geq 0$$

$$\therefore h_1(x)$$

$$\therefore x \in [a, b] \quad h_1(x) \geq 0$$

$$h_2(x) = \int_a^x g(t) dt - x + a$$

$$h_2'(x) = g(x) - 1$$

$$\because 0 \leq g(x) \leq 1 \therefore h_2'(x) \leq 0$$

$$\therefore h_2(x) \quad h_2(a) = 0$$

$$\therefore x \in [a, b] \quad h_2(x) \leq 0$$

$$p(x) = \int_a^x f(u)g(u)du - \int_a^{a+\int_a^x g(t)dt} f(u)du$$

$$p'(x) = f(x)g(x) - f[a + \int_a^x g(t)dt] \cdot g(x) = \left[f(x) - f[a + \int_a^x g(t)dt] \right] g(x)$$

$$\because 0 \leq g(x) \leq 1$$

$$\therefore \int_a^x g(t)dt \leq \int_a^x dt = x - a \therefore a + \int_a^x g(t)dt \leq x$$

$$f(x)$$

$$\therefore f(x) \geq f[a + \int_a^x g(t)dt] \therefore p'(x) \geq 0$$

$$\therefore p(x)$$

$$p(a) = 0 \therefore p(b) \geq 0$$

$$\int_a^b f(x)g(x)dx \geq \int_a^{a+\int_a^b g(t)dt} f(x)dx$$

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$$f(x) = \frac{x}{1+x}, f_1(x) = f(x)$$

$$f_2(x) = f(f_1(x)) = \frac{\frac{x}{1+x}}{1 + \frac{x}{1+x}} = \frac{x}{1+2x}$$

$$f_3(x) = f(f_2(x)) = \frac{\frac{x}{1+2x}}{1 + \frac{x}{1+2x}} = \frac{x}{1+3x}$$

$$f_n(x) = \frac{x}{1+nx}, x \in [0,1]$$

$$S_n = \int_0^1 \frac{x}{1+nx} dx = \frac{1}{n} \int_0^1 \frac{nx+1-1}{1+nx} dx$$

$$= \frac{1}{n} \int_0^1 \left(1 - \frac{1}{1+nx}\right) dx$$

$$= \frac{1}{n} - \frac{1}{n^2} \ln(1+n)$$

$$\lim_{n \rightarrow \infty} n S_n = \lim_{n \rightarrow \infty} n \left[\frac{1}{n} - \frac{1}{n^2} \ln(1+n) \right] = 1 - \lim_{n \rightarrow \infty} \frac{\ln(1+n)}{n}$$

$$= 1$$

$$\frac{\partial f}{\partial y} = 2(y+1)$$

$$f(x, y) = y^2 + 2y + \varphi(x)$$

$$\begin{cases} f(y, y) = (y+1)^2 - (2-y) \\ f(y, y) = y^2 + 2y + \varphi(y) \end{cases}$$

$$\varphi(y) = y - 1$$

$$f(x, y) = y^2 + 2y + x - 1$$

$$f(x, y) = 0 \Rightarrow x = -y^2 - 2y + 1$$

$$V = \int_0^2 \pi (f(x) + 1)^2 dx = \int_0^2 \pi [f^2(x) + 2f(x) + 1] dx = \int_0^2 \pi (2 - x) dx = \pi \left(2x - \frac{x^2}{2} \right) \Big|_0^2 = 2\pi$$

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$$(A) = \begin{pmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & 0 & -3 \end{pmatrix} \xrightarrow{-r_1+r_3} \begin{pmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 4 & -3 & 1 \end{pmatrix} \xrightarrow{-4r_2+r_3} \begin{pmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -3 \end{pmatrix}$$

$$\xrightarrow{\substack{r_3+r_2 \\ -3r_3+r_1}} \begin{pmatrix} 1 & -2 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \end{pmatrix} \xrightarrow{2r_2+r_1} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \end{pmatrix}$$

$$\begin{aligned} x_1 &= -x_4 \\ x_2 &= 2x_4 \\ x_3 &= 3x_4 \\ x_4 &= x_4 \end{aligned} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = c \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} \quad c$$

$$B = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix}$$

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 \\ 0 & 1 & -1 & 1 & \vdots & 0 \\ 1 & 2 & 0 & -3 & \vdots & 0 \end{pmatrix}$$

$$A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 0 \\ 0 & 1 & -1 & 1 & \vdots & 1 \\ 1 & 2 & 0 & -3 & \vdots & 0 \end{pmatrix}$$

$$A \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 \\ 1 & 2 & 0 & -3 & \vdots & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 1 & 2 & 0 & -3 & \vdots & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 4 & -3 & 1 & \vdots & 0 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & \vdots & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 0 & 5 & \vdots & 4 & 12 & -3 \\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1 \\ 0 & 0 & 1 & 3 & \vdots & -1 & -4 & 1 \end{pmatrix} \parallel$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & \vdots & 2 & 6 & -1 \\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1 \\ 0 & 0 & 1 & -3 & \vdots & -1 & -4 & 1 \end{pmatrix} \parallel$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = c_1 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = c_2 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \\ -4 \\ 0 \end{pmatrix} \quad \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = c_3 \begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \parallel$$

$$\therefore B = \begin{pmatrix} -c_1 + 2 & -c_2 + 6 & -c_3 - 1 \\ 2c_1 - 1 & 2c_2 - 3 & 2c_3 + 1 \\ 3c_1 - 1 & 3c_2 - 4 & 3c_3 + 1 \\ c_1 & c_2 & c_3 \end{pmatrix} \parallel$$

$$c_1, c_2, c_3 \parallel$$

$$\parallel \parallel \parallel$$

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \cdots & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \cdots & \mathbf{1} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{1} & \mathbf{1} & \cdots & \mathbf{1} \end{bmatrix} \quad B = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & n \end{bmatrix} \parallel$$

$$|\lambda E - A| = \begin{vmatrix} \lambda - \mathbf{1} & -\mathbf{1} & \cdots & -\mathbf{1} \\ -\mathbf{1} & \lambda - \mathbf{1} & \cdots & -\mathbf{1} \\ \cdots & \cdots & \cdots & \cdots \\ -\mathbf{1} & -\mathbf{1} & \cdots & \lambda - \mathbf{1} \end{vmatrix} = (\lambda - n)\lambda^{n-1} \parallel$$

$$A \quad n \quad \lambda_1 = n, \lambda_2 = \cdots = \lambda_n = 0 \parallel$$

$$A \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix} \quad |\lambda E - B| = \begin{vmatrix} \lambda & 0 & \cdots & 0 & -1 \\ 0 & \lambda & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \lambda - n \end{vmatrix} = (\lambda - n)\lambda^{n-1}$$

$$B \quad n \quad \lambda_1' = n, \lambda_2' = \cdots = \lambda_n' = 0$$

$$|0E - B| = \begin{vmatrix} 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & -n \end{vmatrix}$$

$$r(0E - B) = 1$$

$$B \quad n-1 \quad n-1$$

$$B \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$$A \quad B$$