2014考研数学一真题

一、选择题:1～8小题,每小题4分,共32分.下列每题给出的四个选项中,只有一个选项符合题目要求的,请将所选项前的字母填在答题纸指定位置上.

(1) 下列曲线有渐近线的是 ( )

(A) $y=x+\sin x$ (B) $y=x^2+\sin x$

(C) $y=x+\sin\frac{1}{x}$ (D) $y=x^2+\sin\frac{1}{x}$

(2) 设函数 $f(x)$ 具有二阶导数，$g(x)=f(0)(1-x)+f(1)x$，则在区间 $[0,1]$ 上 ( )

(A) 当 $f'(x)\geq 0$ 时，$f(x)\geq g(x)$ (B) 当 $f'(x)\geq 0$ 时，$f(x)\leq g(x)$

(C) 当 $f''(x)\leq 0$ 时，$f(x)\geq g(x)$ (D) 当 $f''(x)\leq 0$ 时，$f(x)\leq g(x)$

(3) 设 $f(x)$ 是连续函数，则 $\int_0^1 dy\int_{-\sqrt{1-y^2}}^{1-y} f(x,y)dx=$ ( )

(A) $\int_0^1 dx\int_0^{x-1} f(x,y)dy+\int_{-1}^0 dx\int_0^{\sqrt{1-x^2}} f(x,y)dy$

(B) $\int_0^1 dx\int_0^{1-x} f(x,y)dy+\int_{-1}^0 dx\int_{-\sqrt{1-x^2}}^0 f(x,y)dy$

(C) $\int_0^{\frac{\pi}{2}} d\theta\int_0^{\frac{1}{\cos\theta+\sin\theta}} f(r\cos\theta,r\sin\theta)dr+\int_{\frac{\pi}{2}}^{\pi} d\theta\int_0^1 f(r\cos\theta,r\sin\theta)dr$

(D) $\int_0^{\frac{\pi}{2}} d\theta\int_0^{\frac{1}{\cos\theta+\sin\theta}} f(r\cos\theta,r\sin\theta)rdr+\int_{\frac{\pi}{2}}^{\pi} d\theta\int_0^1 f(r\cos\theta,r\sin\theta)rdr$

(4) 若 $\int_{-\pi}^{\pi}(x-a_1\cos x-b_1\sin x)^2dx=\min\limits_{a,b\in R}\left\{\int_{-\pi}^{\pi}(x-a\cos x-b\sin x)^2dx\right\}$，则

$a_1\cos x+b_1\sin x=$ ( )

(A) $2\sin x$ (B) $2\cos x$ (C) $2\pi\sin x$ (D) $2\pi\cos x$

(5) 行列式 $\begin{vmatrix} 0 & a & b & 0 \\ a & 0 & 0 & b \\ 0 & c & d & 0 \\ c & 0 & 0 & d \end{vmatrix} =$ ( )

(A) $(ad-bc)^2$　　(B) $-(ad-bc)^2$　　(C) $a^2d^2-b^2c^2$　　(D) $b^2c^2-a^2d^2$

(6) 设 $\alpha_1,\alpha_2,\alpha_3$ 是3维向量，则对任意常数 k,l，向量 $\alpha_1+k\alpha_3,\ \alpha_2+l\alpha_3$ 线性无关是向量 $\alpha_1,\alpha_2,\alpha_3$ 线性无关的 ( )

(A) 必要非充分条件　　(B) 充分非必要条件

(C) 充分必要条件　　(D) 既非充分又非必要条件

(7) 设随机事件 A 与 B 相互独立，且 $P(B)=0.5$，$P(A-B)=0.3$，则 $P(B-A)=$ ( )

(A)0.1　　(B)0.2　　(C)0.3　　(D)0.4

(8) 设连续性随机变量 X_1 与 X_2 相互独立，且方差均存在，X_1 与 X_2 的概率密度分别为 $f_1(x)$ 与 $f_2(x)$，随机变量 Y_1 的概率密度为 $f_{Y_1}(y)=\frac{1}{2}[f_1(y)+f_2(y)]$，随机变量 $Y_2=\frac{1}{2}(X_1+X_2)$，则 ( )

(A) $EY_1>EY_2,\ DY_1>DY_2$　　(B) $EY_1=EY_2,\ DY_1=DY_2$

(C) $EY_1=EY_2,\ DY_1<DY_2$　　(D) $EY_1=EY_2,\ DY_1>DY_2$

二、填空题：9～14小题，每小题4分，共24分.请将答案写在答题纸指定位置上.

(9) 曲面 $z=x^2(1-\sin y)+y^2(1-\sin x)$ 在点 $(1,0,1)$ 处的切平面方程为________

(10) 设 $f(x)$ 是周期为4的可导奇函数，且 $f'(x)=2(x-1)$，$x\in[0,2]$，则 $f(7)=$________

(11) 微分方程 $xy'+y(\ln x-\ln y)=0$ 满足条件 $y(1)=e^3$ 的解为 $y=$________

(12) 设 L 是柱面 $x^2+y^2=1$ 与平面 $y+z=0$ 的交线，从 z 轴正向往 z 轴负向看去为逆时针方向，则曲面积分 $\oint_L zdx+ydz=$________

[redacted]

(20)(本题满分 11 分)设 $A=\begin{pmatrix}1 & -2 & 3 & -4\\ 0 & 1 & -1 & 1\\ 1 & 2 & 0 & -3\end{pmatrix}$，E 为 3 阶单位矩阵.

（I）求方程组 $A\boldsymbol{x}=\boldsymbol{0}$ 的一个基础解系；　（II）求满足 $\boldsymbol{AB}=\boldsymbol{E}$ 的所有矩阵 B.

(21)(本题满分 11 分) 证明 n 阶矩阵 $\begin{pmatrix}1 & 1 & \cdots & 1\\ 1 & 1 & \cdots & 1\\ \vdots & \vdots & & \vdots\\ 1 & 1 & \cdots & 1\end{pmatrix}$ 与 $\begin{pmatrix}0 & \cdots & 0 & 1\\ 0 & \cdots & 0 & 2\\ \vdots & & \vdots & \vdots\\ 0 & \cdots & 0 & n\end{pmatrix}$ 相似.

(22)(本题满分 11 分)

设随机变量 X 的概率分布为 $P(X=1)=P(X=2)=\dfrac{1}{2}$，在给定 $X=i$ 的条件下，随机变量 Y 服从均匀分布 $U(0,\ i)(i=1,2)$.

（I）求 Y 的分布函数 $F_Y(y)$；

（II）求 $E(Y)$.

(23)(本题满分 11 分)

设总体 X 的分布函数为 $F(x,\ \theta)=\begin{cases}1-e^{-\frac{x^2}{\theta}}, & x\geq 0\\ 0, & x<0\end{cases}$，其中 θ 是未知参数且大于零. X_1、X_2、…、X_n 为来自总体 X 的简单随机样本.

（I）求 $E(X)$ 与 $E(X^2)$；

（II）求 θ 的最大似然估计量 $\hat{\theta}_n$；

（III）是否存在实数 a，使得对任何 $\varepsilon>0$，都有 $\lim\limits_{n\to\infty}P\left\{\left|\hat{\theta}_n-a\right|\geq\varepsilon\right\}=0$？

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9 $2x-y-z-1=0$ 10 1

11 $y=xe^{2x+1}$ 12 **π**

13 $[-2,2]$ $\frac{2}{5n}$

15

$$\lim_{x\to\infty}\frac{\int_1^x(\mathrm{t}^2(\mathrm{e}^{\frac{1}{t}}-1)-\mathrm{t})\,\mathrm{dt}}{x^2\ln(1+\frac{1}{x})}=\lim_{x\to\infty}\frac{\int_1^x(\mathrm{t}^2(\mathrm{e}^{\frac{1}{t}}-1)-\mathrm{t})\,\mathrm{dt}}{x^2\cdot\frac{1}{x}}=\lim_{x\to\infty}\frac{x^2(\mathrm{e}^{\frac{1}{x}}-1)-x}{1}=\lim_{x\to\infty}x^2(\mathrm{e}^{\frac{1}{x}}-1-\frac{1}{x})$$

$$\underline{\underline{\frac{1}{x}=t}}\lim_{x\to\infty}\frac{e^t-1-t}{t^2}=\lim_{x\to\infty}\frac{1+t+\frac{1}{2}t^2+o(\mathrm{t}^2)-1-\mathrm{t}}{t^2}=\frac{1}{2}$$

16.

$y^3+xy^2+x^2y+6=0(1)$

(1) x

$(3y^2+2xy+x^2)y'+y^2+2xy=0(2)$

$y'=0$, $y=0$ $y=-2x$

$y=-2x$ **(1)** $x=1,y=-2$

2 x

$(6yy'+2y+2xy'+2x)y'+(3y^2+2xy+x^2)y''+2yy'+2y+2xy'=0$

$x=1,y=-2,y'\big|_{x=1}=0,\quad y''\big|_{x=1}=\frac{4}{9}>0$

$x=1$ **,**$f(x)$ $f(1)=-2$

$$\frac{\partial z}{\partial x} = f' \cdot e^x \cdot \cos y,$$

$$\frac{\partial^2 z}{\partial x^2} = \cos y \cdot (f'' \cdot e^x \cdot \cos y \cdot e^x + f' \cdot e^x) = f'' \cdot (e^x \cdot \cos y)^2 + f' \cdot e^x \cdot \cos y$$

$$\frac{\partial z}{\partial y} = f' \cdot e^x \cdot (-\sin y),$$

$$\frac{\partial^2 z}{\partial y^2} = -e^x[f'' \cdot e^x \cdot (-\sin y) + f' \cdot \cos y] = (e^x)^2 \sin y^2 f'' - f' \cdot \cos y \cdot e^x$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = f'' \cdot e^{2x} = (4z + e^x \cdot \cos y)e^{2x}$$

$$\therefore f''(e^x \cdot \cos y) = 4f(e^x \cdot \cos y) + e^x \cdot \cos y$$

$$t = e^x \cdot \cos y, \therefore f''(t) = 4f(t) + t$$

$$\therefore y'' - 4y = x$$

$$\lambda^2 - 4 = 0 \quad \therefore x = \pm 2 \quad \therefore \widetilde{y(x)} = C_1 e^{2x} + C_2 e^{-2x}$$

$$y^* = (ax + b) \qquad \therefore y^* = -\frac{1}{4}x$$

$$\therefore y = \widetilde{y(x)} + y^* = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{4}x$$

$$y(0)=0=C_1 + C_2$$

$$y'(0)=0=x_1 - x_2 - \frac{1}{4}$$

$$\therefore \begin{cases} C_1 + C_2 = 0 \\ 2C_1 - 2C_2 = \frac{1}{4} \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{16} \\ C_2 = -\frac{1}{16} \end{cases}$$

$$\therefore f(\mu) = \frac{1}{16}e^{2\mu} - \frac{1}{16}e^{-2\mu} - \frac{1}{4}\mu$$

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$$\Sigma_1 : \begin{cases} x^2 + y^2 \le 1 \\ z = 1 \end{cases}$$

$(x \quad 1)^3 \qquad (y \quad 1)^3 d \qquad (= \qquad ($

1 1

$$=\iiint_{\Omega}(3x^2+3y^2+7)\,dxdydz=\int_0^1 dz\iint_{D(z)}(3x^2+3y^2+7)dxdy$$

$$=\int_0^1 dz\int_0^{2\pi}d\theta\int_0^{\sqrt{z}}(3r^2+7)rdr=4\pi$$

$\iiint_{\Omega}(6x+6y)\,dxdydz=0$ $xoz,\ yoz$

$f(x,y)=6x+6y$ y,x

Σ_1 $\iint_{\Sigma_1}(x-1)^3dydz+(y-1)^3dzdx+(z-1)\,dxdy=0$

$$\oiint_{\Sigma}(x-1)^3dydz+(y-1)^3dzdx+(z-1)\,dxdy=-4\pi$$

$\because \cos a_n - a_n = \cos b_n$

$\therefore a_n = \cos a_n - \cos b_n$

$0<a_n<\dfrac{\pi}{2}$

$\therefore \cos a_n - \cos b_n > 0$

$\Rightarrow a_n < b_n$

$\because a_n>0, b_n>0,$ $\sum_{n=1}^{\infty}b_n$

$\therefore \sum_{n=1}^{\infty}a_n$

$\lim_{n\to\infty}a_n=0$

$$\frac{a_n}{b_n}=\frac{\cos a_n-\cos b_n}{b_n}$$

$$=\frac{-2\sin\frac{a_n+b_n}{2}\sin\frac{a_n-b_n}{2}}{\frac{a_n+b_n}{2}\cdot\frac{a_n-b_n}{2}}\cdot\frac{a_n^2-b_n^2}{4b_n}$$

$$\le\frac{b_n^2-a_n^2}{2b_n}$$

$$\le\frac{b_n^2}{2b_n}=\frac{b_n}{2}$$

$$\because 0<a_n<\frac{\pi}{2},0<b_n<\frac{\pi}{2}$$

$$\sum_{n=1}^{+\infty}b_n$$

$$\therefore\sum_{n=1}^{\infty}\frac{a_n}{b_n}$$

20.

$$(A)=\begin{pmatrix}1&-2&3&-4\\0&1&-1&1\\1&2&0&-3\end{pmatrix}\xrightarrow{-r_1+r_3}\begin{pmatrix}1&-2&3&-4\\0&1&-1&1\\0&4&-3&1\end{pmatrix}\xrightarrow{-4r_2+r_3}\begin{pmatrix}1&-2&3&-4\\0&1&-1&1\\0&0&1&-3\end{pmatrix}$$

$$\xrightarrow[-3r_3+r_1]{r_3+r_2}\begin{pmatrix}1&-2&0&5\\0&1&0&-2\\0&0&1&-3\end{pmatrix}\xrightarrow{2r_2+r_1}\begin{pmatrix}1&0&0&1\\0&1&0&-2\\0&0&1&-3\end{pmatrix}$$

$$\begin{matrix}x_1=-x_4\\x_2=2x_4\\x_3=3x_4\\x_4=x_4\end{matrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=c\begin{pmatrix}-1\\2\\3\\1\end{pmatrix}\ \text{c}$$

$$B=\begin{pmatrix}x_1&y_1&z_1\\x_2&y_2&z_2\\x_3&y_3&z_3\end{pmatrix}$$

$$A\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}\Rightarrow\begin{pmatrix}1&-2&3&-4&\vdots&1\\0&1&-1&1&\vdots&0\\1&2&0&-3&\vdots&0\end{pmatrix}$$

$$A\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\Rightarrow\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 0 \\ 0 & 1 & -1 & 1 & \vdots & 1 \\ 1 & 2 & 0 & -3 & \vdots & 0 \end{pmatrix}$$

$$A\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\Rightarrow\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 \\ 1 & 2 & 0 & -3 & \vdots & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 1 & 2 & 0 & -3 & \vdots & 0 & 0 & 1 \end{pmatrix}\to\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 4 & -3 & 1 & \vdots & 0 & 0 & 1 \end{pmatrix}$$

$$\to\begin{pmatrix} 1 & -2 & 3 & -4 & \vdots & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & \vdots & 0 & 0 & 1 \end{pmatrix}\to\begin{pmatrix} 1 & -2 & 0 & 5 & \vdots & 4 & 12 & -3 \\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1 \\ 0 & 0 & 1 & 3 & \vdots & -1 & -4 & 1 \end{pmatrix}$$

$$\to\begin{pmatrix} 1 & 0 & 0 & 1 & \vdots & 2 & 6 & -1 \\ 0 & 1 & 0 & -2 & \vdots & -1 & -3 & 1 \\ 0 & 0 & 1 & -3 & \vdots & -1 & -4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}=c_1\begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix}+\begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \end{pmatrix}\quad\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}=c_2\begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix}+\begin{pmatrix} 6 \\ -3 \\ -4 \\ 0 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}=c_3\begin{pmatrix} -1 \\ 2 \\ 3 \\ 1 \end{pmatrix}+\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\therefore B=\begin{pmatrix} -c_1+2 & -c_2+6 & -c_3-1 \\ 2c_1-1 & 2c_2-3 & 2c_3+1 \\ 3c_1-1 & 3c_2-4 & 3c_3+1 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

c_1, c_2, c_3

$$A=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} B=\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & n \end{bmatrix}$$

$$|\lambda E - A| = \begin{vmatrix} \lambda-1 & -1 & \cdots & -1 \\ -1 & \lambda-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & \lambda-1 \end{vmatrix} = (\lambda - n)\lambda^{n-1}$$

A n λ_1=n, $\lambda_2 = \cdots = \lambda_n$=0

A A

$$A \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix} \quad |\lambda E - B| = \begin{vmatrix} \lambda & 0 & \cdots & 0 & -1 \\ 0 & \lambda & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \lambda - N \end{vmatrix} = (\lambda - n)\lambda^{n-1}$$

B n $\lambda_1{}' $=n, $\lambda_2{}' = \cdots = \lambda_n{}'$=0

$$|0E - B| = \begin{vmatrix} 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & \cdots & 0 & -2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & -n \end{vmatrix}$$

$r(0E - B) = 1$

B n-1 n-1

B $B \sim \begin{bmatrix} n & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$

A B

$$\begin{aligned} F_y(y) &= P(Y \le y) \\ &= P(Y \le y, X = 1) + P(Y \le y, X = 2) \\ &= P(Y \le y \mid X = 1)P(X = 1) + P(Y \le y \mid X = 2)P(X = 2) \\ &= \frac{1}{2}P(Y \le y \mid X = 1) + \frac{1}{2}P(Y \le y \mid X = 2) \end{aligned}$$

$y < 0$ $F_Y(y) = 0$

$0 \le y < 1$ $F_Y(y) = \frac{1}{2}y + \frac{1}{2} \times \frac{1}{2}y = \frac{3}{4}y$

$$1 \le y < 2 \qquad F_Y(y) = \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} y = \frac{1}{2} + \frac{y}{4}$$

$$y \ge 2 \qquad F_Y(y) = \frac{1}{2} + \frac{1}{2} = 1$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{3}{4} y & 0 \le y < 1 \\ \frac{1}{2} + \frac{y}{4} & 1 \le y < 2 \\ 1 & y \ge 2 \end{cases}$$

$$f_Y(y) = F'_Y(y) = \begin{cases} \frac{3}{4} & 0 < y < 1 \\ \frac{1}{4} & 1 \le y < 2 \\ 0 & \end{cases}$$

$$EY = \int_{-\infty}^{+\infty} y f_Y(y)\,dy = \frac{3}{4}\int_0^1 y dy + \frac{1}{4}\int_1^2 y dy$$
$$= \frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{3}{2} = \frac{3}{4}$$

$$F(x;\theta) = \begin{cases} 1 - e^{-\frac{x^2}{\theta}} & x \ge 0 \\ 0 & \end{cases}$$

$$f(x;\theta) = F'(x;\theta) = \begin{cases} \frac{2X}{\theta} e^{-\frac{x^2}{\theta}} & x \ge 0 \\ 0 & \end{cases}$$

$$EX = \int_{-\infty}^{+\infty} x f(x;\theta)\,dx = \int_0^{+\infty} x \frac{2x}{\theta} e^{-\frac{x^2}{\theta}} dx = -\int_0^{+\infty} x de^{-\frac{x^2}{\theta}} = -xe^{-\frac{x^2}{\theta}}\Big|_0^{+\infty} + \int_0^{+\infty} e^{-\frac{x^2}{\theta}} dx = \sqrt{\pi\theta}$$

$$EX^2 = \int_{-\infty}^{+\infty} x^2 f(x;\theta)\,dx = \int_0^{+\infty} x^2 \frac{2x}{\theta} e^{-\frac{x^2}{\theta}} dx \underline{\underline{\;x^2 = t\;}} \int_0^{+\infty} t \frac{2\sqrt{t}}{\theta} e^{-\frac{t}{\theta}} \frac{1}{2\sqrt{t}} dt = \int_0^{+\infty} t \frac{1}{\theta} e^{-\frac{t}{\theta}} dt = \theta$$

$x_1, x_2, \cdots, x_n$

$$L(\theta) = \prod_{i=1}^{n} f(x_{ij}\ \theta) = \begin{cases} \dfrac{2^n \prod_{i=1}^{n} x_i}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^2} & x_i \geq 0 (i = 1, 2, \cdots, n) \\ 0 & \end{cases}$$

$x_i \geq 0\ \ i=1\ \ 2\ \cdots\ \ n$

$$L(\theta) = \frac{2^n \prod_{i=1}^{n} x_i}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i^2}$$

$$\ln L(\theta) = n \ln 2 + \ln \prod_{i=1}^{n} x_i - n \ln \theta - \frac{1}{\theta} \sum_{i=1}^{n} x_i^2$$

$$\frac{d \ln L(\theta)}{d\theta} = -\frac{n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} x_i^2 = 0$$

$$\Rightarrow \hat{\theta}_n = \frac{1}{n} \sum_{i=1}^{n} x_i^2 \quad \theta$$

$\because x_1, x_2, \cdots, x_n \qquad \therefore x_1^2, x_2^2, \cdots, x_n^2$

$EX_i^2 = \theta (i = 1, 2, \cdots, n)$

$$\lim_{n\to\infty} P\left\{\left|\frac{1}{n}\sum_{i=1}^{n} x_i^2 - E\frac{1}{n}\sum_{i=1}^{n} x_i^2\right| < \varepsilon\right\} = 1$$

$$E\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \frac{1}{n}\sum_{i=1}^{n} Ex_i^2 = \theta$$

$\therefore \qquad a=\theta, \qquad st. \qquad \forall \varepsilon > 0,$

$$\lim_{n\to\infty} P\left\{\left|\hat{\theta}_n - a\right| < \varepsilon\right\} = 1$$